

# Incentive contracts when agents distort probabilities\*

Victor Gonzalez-Jimenez<sup>†</sup>

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## Abstract

I propose a novel class of incentive contracts designed to take advantage of the regularity that individuals distort probabilities. The proposed contracts introduce risk in the agent's compensation and allow the principal to adjust the degree of risk faced by the agent. The latter feature permits the principal to target probabilities that enhance the motivation to perform the task. A theoretical framework and an experiment demonstrate that the proposed contracts can generate higher performance than traditional contracting modalities. I show that probability distortions caused by likelihood insensitivity—cognitive limitations that restrict the accurate evaluation of probabilities—generate these findings.

**JEL Classification :** C91, C92, J16, J24.

**Keywords:** Contracts, Risk Attitudes, Incentives, Probability Weighting, Experiments.

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<sup>†</sup>University of Vienna, Department of Economics and Vienna Center for Experimental Economics, Oskar-Morgernstern-Platz 1. 1090 AT, Vienna, Austria. E-mail: [victor.gonzalez@univie.ac.at](mailto:victor.gonzalez@univie.ac.at).

# 1. Introduction

The traditional focus of contract theory has been the optimal implementation of monetary incentives in settings of asymmetric information. A standard result from this literature is that a principal, who lacks knowledge about the agents' abilities on the delegated task, can profitably incentivize labor supply with a menu of contracts offering an array of carefully designed monetary transfers (Laffont and Tirole, 1993, Myerson, 1981, Green and Laffont, 1977). Recent literature that explores the influence of behavioral biases on contract design shows that simple and costless extensions to these standard optimal contracts can further enhance labor supply. For instance, contracts in which production goals are specified can enhance motivation when agents exhibit reference-dependence preferences (Corgnet et al., 2018, 2015, Gómez-Miñambres, 2012) and/or hyperbolic discounting (Kaur et al., 2015).<sup>1</sup>

In this paper, I introduce a contracting modality designed to incentivize agents who are prone to probability distortion. Empirical evidence from the literature of decision-making shows that individuals, when making decisions under risk, tend to overweight small probabilities and underweight medium to large probabilities (Abdellaoui, 2000, Gonzalez and Wu, 1999, Wu and Gonzalez, 1996, Tversky and Fox, 1995, Tversky and Kahneman, 1992). With the class of contracts proposed in this paper, the principal not only activates these probability distortions but also targets those that enhance labor supply the most. Since these incentives are absent in traditional contracting modalities, the contract proposed in this paper has the potential to generate greater output at no extra cost for the principal.

In the proposed contract, henceforth called *probability contract*, the principal can introduce uncertainty in the agent's payment. Specifically, the agent is incentivized with an incentive scheme that obtains one of two possible outcomes: a monetary compensation that depends on individual performance on the delegated task, and a lump sum payment that does not depend on performance. The distinctive feature of this contract is that the principal can determine the likelihood that the performance-contingent compensation is paid, and this decision is made before the contract is signed and, thus, before the task is performed. In a setting with full commitment, that is a setting whereby the principal credibly commits to pay the outcomes specified in the contract and to respect the stochastic rule that determines these outcomes, the agents' decision about how much output to supply not only depends on the monetary incentives offered by the contract, but also, and more importantly, on their perceived probability that performance on the delegated task affects their compensation.

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<sup>1</sup>Other examples of these simple extensions are: including contests for status in the organization when individuals exhibit a preference for status (Besley and Ghatak, 2008, Auriol and Renault, 2008, Moldovanu et al., 2007), and credit contracts with heavy front loaded repayment and large penalties for delaying repayment which take advantage of time-inconsistent and naive agents (Heidhues and Koszegi, 2010).

To understand how the probability contract can outperform traditional deterministic contracts, consider a setting in which both class of contracts are cost-equivalent for the principal. That is, the *expected* monetary compensation associated to supplying any level of output when agents work under the probability contract is equal to the monetary compensation given to agents when the same level of output is supplied under a traditional contract. Expected value maximizers will be equally motivated under both contracts and, as a result, will work to supply the same amount of output. However, when the assumption that agents perceive probabilities accurately is relaxed, and instead it is assumed that they overweight the probability that the performance-contingent outcome realizes, the probability contract motivates agents to supply more output. The underlying reason for such boost in labor supply is that this distortion of probabilities inflates the agents' perceived benefits of supplying higher levels of effort.

A simple theoretical framework serves two purposes. First, it presents the necessary conditions guaranteeing the main result of the paper, which is that probability contracts generate higher output than more traditional contracts at no extra cost for the principal. Second, it provides a set of predictions that are empirically tested with a laboratory experiment. The theoretical conditions guaranteeing the main result of the paper are that: i) the representation of the agents' risk preferences admits probability distortions and ii) the principal implements these contracts with a probability that is strongly overweighted by agents, with the aim of inducing risk seeking attitudes in the agent and, consequently, a preference for stochastic compensation schemes. I ensure condition i) by characterizing the agents' risk preferences with rank-dependent utility (Quiggin, 1982). Hence, in this paper probability distortions are modeled by means of probability weighting functions that transform cumulative probabilities. Condition ii) depends on the shape of the agents' probability weighting function. Specifically, weighting functions with global concavity or with an inverse-S shape generate intervals wherein probabilities are overweighted by the agent. The existence of these intervals along with a novel technical condition that I call *bounded concavity of the weighting function*—implying that positive changes in probability translate into a positive but finite changes in the individual's marginal sensitivity to probabilities—ensure the existence of probabilities that, when targeted by the principal, enhance labor supply. Failing to comply with any of these conditions yields the opposite result: probability contracts, when introducing uncertainty in the agent's compensation, generate lower output than traditional contracts. These antagonistic results, generated by complying or not with these conditions, develop into the main testable predictions of the model.

The theoretical model also introduces and formalizes a taxonomy of probability distortions taken from the literature of decision theory (Abdellaoui et al., 2011, Wakker, 2010, Tversky

and Wakker, 1995). In particular, I classify probability distortions as caused either by motivational factors, namely *optimism* or *pessimism* toward risk, or by *likelihood insensitivity*, which refers to the cognitive inability of individuals to accurately evaluate probabilities (Tversky and Wakker, 1995). I demonstrate that optimism or likelihood insensitivity suffice to guarantee the efficiency of the proposed contracting modality. Thus, when implementing the probability contract, principals can exploit probability distortions originating from agents' optimism toward risk or those emerging from the agents' cognitive limitations to perceive probabilities.

A controlled laboratory experiment demonstrates that the proposed contracts, when implemented with probability  $p = 0.10$ , yield higher performance in an effort intensive task as compared to a cost-equivalent piece-rate. In contrast, I find that probability contracts implemented with larger probabilities, namely  $p = 0.30$  or  $p = 0.50$ , yield no differences in performance as compared to the cost-equivalent piece-rate. The experiment also features an elicitation of the utility and probability weighting functions of subjects. These data show that subjects display an average weighting function with a strong inverse-S shape and considerable optimism. I demonstrate that this pattern of probability distortion explains the treatment effects found in the effort task. In addition, the data demonstrate that likelihood insensitivity, and not optimism, explains the documented difference in performance between the probability contract implemented with  $p = 0.10$  and the cost-equivalent piece-rate.

While the proposed contract seems abstract, its incentives can be brought to practice using standard tools of personnel economics. For instance, the principal can implement the incentives of the probability contract using compensation schemes that include monetary bonuses that are paid in the contingency that a milestone is reached. When the principal is able to set the milestone, allowing her to specify the probability that the bonus is achieved by the agent, and when she is able to adjust the magnitude of the bonus, the beneficial incentives of the contracts proposed in this paper apply. I provide a detailed explanation of this application and provide some more in the last section of the paper.

This paper contributes to several strands of literature. Its theoretical and empirical results add in multiple ways to the literature of behavioral contract theory (See Koszegi (2014) for a review). The main contribution to this literature is the result that, when agents distort probabilities, stochastic contracts allowing the principal to determine the degree of risk faced by the agent are preferable to deterministic contracts. This is at odds with standard results from the theory of incentives stating that the principal faces a trade-off between incentives and insurance. With the proposed contract modality, the principal can provide incentives by exposing the agent to high risk. While the optimality of stochastic contracts has been put forward in other settings, such as multitasking environments (Ederer et al., 2018), when

agents exhibit aspiration levels (Haller, 1985), or when agents are loss averse (Corgnet and Hernán-González, 2019, Herweg et al., 2010), I am the first to theoretically and empirically show that they are desirable when agents exhibit probability distortions.

To my knowledge only Spalt (2013) has studied optimal contract design under probability weighting. The most relevant distinction with respect to that paper is that I focus on the agents' incentive compatibility constraint. That is, I study the incentives that result from offering the proposed contracting modality with different degrees of risk and establish the optimal degree of risk to be faced by the agent. Spalt's (2013) analysis does not consider these incentives and ignores such constraint. Another relevant difference is that I propose a general type of contracts that introduces risk in the agents' compensation. They can be brought to practice in multiple ways, among which, but not only restricted to, compensation plans with stock options.<sup>2</sup>

Furthermore, some theoretical studies show that in settings of moral hazard the principal can derive rents when she contracts with optimistic agents (De La Rosa, 2011, Gervais et al., 2011, Santos-Pinto, 2008). The rationale is that optimistic agents are willing to accept contracts that pay high-powered incentives in the unlikely contingency that favorable outcomes realize and also excessively low-powered incentives when likely but unfavorable outcomes realize. This paper demonstrates theoretically and empirically that optimism is a sufficient but not a necessary condition to ensure the efficiency of the proposed contract. That is because in the absence of optimism but in the presence of likelihood insensitivity—the cognitive component of probability weighting—the existence of a region of probabilities that the principal can profitably target with the probability contract is guaranteed. Thus, likelihood insensitivity emerges as a novel deviation from standard preferences that can be exploited by the principal and that is complementary to optimism. Another relevant distinction with respect to these studies is that optimism and pessimism are modeled as probability distortions generated by concave and convex probability weighting functions, respectively. Therefore, optimism and pessimism can emerge without the assumption, common in those

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<sup>2</sup>Other crucial differences with respect to Spalt (2013) are the following. First, Spalt (2013) demonstrates the profitability of compensation plans with stock options using a calibration exercise that employs parameters estimated in classical experiments. Instead, I demonstrate theoretically, that is with analytical solutions, that the type of stochastic contracts that I propose is more efficient than more traditional contracting modalities. Also, my experiment is designed to directly link the subjects' performance under probability contracts to their risk preferences. This experimental design feature allows me to cleanly establish whether the subjects' probability weighting, and not other factors, drives the result that the proposed contracts can generate greater performance. Second, I use a well-known decomposition of probability distortions. This decomposition allows me to examine the role of the motivational and cognitive factors of probability distortion on the effectiveness of the stochastic contract that is proposed. Not only is Spalt (2013) silent about this distinction, but his results are restricted to one family of probability weighting functions, namely that proposed by Tversky and Kahneman (1992), which cannot separate these components limiting his analysis of probability weighting on contract design.

papers, that the agent holds erroneous beliefs about her ability or productivity. Finally, the chosen characterization of optimism and pessimism is not only descriptively desirable, as it for instance can accommodate anomalies and regularities of risky decision-making such as the well-known “Allais paradox”, but it is also endowed with the preference foundations of rank-dependence (Abdellaoui, 2002, Quiggin, 1982) which provide a general and axiomatized framework of the notions of pessimism and optimism.<sup>3</sup>

Finally, the results of this study also contribute to the literature of decision-theory in multiple ways. To my knowledge I am the first to provide applications of probability weighting elicitation techniques to the context of incentives. Furthermore, the experimental results illustrate the importance of using parametrized probability weighting functions that can separate likelihood insensitivity from optimism. I use the different methods proposed by Wakker (2010) and applied in Abdellaoui et al. (2011) to isolate these two components of probability weighting, and show that they contribute unequally to the effectiveness of the contract. Finally, the model highlights the importance of some properties of the curvature of the weighting function that are often left unstudied. Specifically, I show that agents who exhibit *bounded concavity of the weighting function*, a condition imposing a boundary to the degree of concavity that a weighting function can display, and who also overweight some non-empty interval of probability, exhibit, regardless of the concavity of their utility function, a region of probabilities that can be profitably targeted by the principal with the proposed contract.

## 2. The model

Consider a principal (she) who delegates a task to a representative agent (he). The agent’s decision consists on exerting a level of effort  $e \in [0, \bar{e}]$  on the task. This decision depends on the disutility associated to exerting effort, as well as on the monetary incentives included in a take-it or leave-it contract that is offered by the principal.

I assume that the agent experiences marginally increasing disutility with higher levels of effort. Specifically, the disutility from effort is represented by the cost function  $c(e)$  a continuously differentiable, strictly increasing, and convex function. The properties of the cost function are formally defined in the following assumption:

**Assumption 1.**  $c(e)$  is a  $\mathcal{C}^2$  function with  $c' > 0$ ,  $c'' > 0$ , and  $c(0) = 0$ .

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<sup>3</sup>In addition, I show that the principal can use probability contracts to exploit optimistic agents through a completely different channel. Specifically, the stochastic nature of these contracts ensures that optimistic agents inflate the expected returns of supplying high levels of output, even when the agents’ expected pay is kept constant for all the possible probabilities that the principal can specify.

Moreover, it is assumed that the effort level chosen by the agent translates into output in a deterministic way, according to the following production function:

**Assumption 2.**  $y = Q(\theta, e) = \theta e$  for all  $\theta \in [0, 1]$ .

The parameter  $\theta \in [0, 1]$ , included in the production function, captures the agent’s ability on the task. Hence, Assumption 2 states that agents with higher ability deliver can higher levels of output as compared to agents with lower ability without having to exert higher effort.

Settings whereby the link between effort, output, and ability is deterministic have been labeled as “false moral hazard” in the literature (See (Laffont and Martimort, 2002) Ch. 7.2). While in these models the principal can directly observe output, such observation does not allow her to disentangle the agent’s ability from his choice of effort. For instance, an agent with low ability can, by means of exerting high effort, make up for his lack of ability. This difficulty makes this type of models similar in spirit to adverse selection models. The rationale for investigating the effectiveness of the proposed contract in this a setting, rather than using a standard moral hazard environment, is that with this type of model it can be established whether introducing uncertainty in the agent’s problem using the probability contract is beneficial in a setting in which he otherwise would not have to face uncertainty.

To incentivize the agent to exert effort, the principal offers the agent a contract including a transfer  $t$ . I assume that the monetary incentives included in the contract enter the agent’s utility through the function  $b(t)$  about which I make the following assumption:

**Assumption 3.**  $b(t)$  is a  $\mathcal{C}^2$  function with  $b(0) = 0$  and  $b' > 0$ .

Note that Assumption 3 does not impose restrictions on the sign of the second derivative of  $b$ . That is because the results of the model will be evaluated under the two signs that this derivative attains.

We study two types of contracts: deterministic contracts and the special type of stochastic contracts proposed in this paper. As a benchmark of deterministic contracts, I use piece-rates. Formally, a piece-rate is a compensation schedule  $t_d := ay$ , where  $a > 0$  represents a monetary quantity. These contracts have the property that equally-sized increments of output taking place at any point in the output space are rewarded with the same monetary compensation. Piece-rates in settings of adverse selection can constitute an optimal menu of contracts (Gibbons, 1998). That is because they induce the agent to choose the optimal amount of effort given his ability, via the *taxation principle* (Guesnerie, 1995).<sup>4</sup>

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<sup>4</sup>Specifically, piece-rates can compose a menu of linear contracts forming the upper envelope of the optimal incentive contract obtained through the truthful revelation mechanism. Thus, an agent of ability  $\theta$  self-selects to the best-fitting linear contract by choosing  $y(\theta)$  that entails receiving a wage  $t(y(\theta))$ .

All in all, the agent’s utility when offered the contract  $t_d$  is:

$$U(t_d) = b(ay) - c(e). \tag{1}$$

Alternatively, the principal can incentivize the agent to work on the task using the probability contract. This special type of stochastic contract also offers a monetary compensation that depends on the agent’s level of supplied output, but, unlike the piece-rate, this performance-contingent compensation is not given with certainty. Instead, the agent receives such compensation with a probability  $p \in (0, 1]$  that is ex-ante chosen by the principal and is known by the agent. As a consequence, the principal has two channels to motivate the agent with the probability contract: i) through the monetary rewards given in exchange of the level of output that is supplied and ii) through changes in the likelihood that such rewards are indeed paid.

Formally, the probability contract consists of the lottery-like compensation schedule  $t_s := (Ay, p; 0, 1 - p)$ , where  $A > 0$  represents a monetary quantity.<sup>5</sup> The underlying timing of this type of contracts is as follows. The principal moves first choosing  $p \in (0, 1]$ . After this choice is made,  $p$  is communicated to the agent before he makes a decision about the level of  $e$  to be exerted and the level  $y$  to be supplied. Next, the agent chooses  $e$ . Finally, when the contracted work-span concludes, a random device to which the principal credibly commits determines whether or not the agent’s compensation depends on the supplied level of  $y$ .

I assume that the agent’s risk preferences are characterized by rank-dependent utility (RDU, henceforth) (Quiggin, 1982). These risk preferences are descriptively accurate inasmuch as they accommodate the abundant evidence that individuals systematically distort probabilities (Abdellaoui, 2000, Gonzalez and Wu, 1999, Wu and Gonzalez, 1996, Tversky and Fox, 1995, Tversky and Kahneman, 1992). Moreover, this representation of the agent’s risk preferences entails that risk attitude is not only determined by the agent’s sensitivity to the monetary outcomes offered by a contract, captured by the curvature of  $b$ , but also by the agent’s sensitivity to the probabilities associated to these outcomes. Sensitivity to probabilities is captured in the model by a probability weighting function,  $w(p)$  which transforms probabilities. I make the following assumptions on  $w(p)$ :

**Assumption 4.** *A probability weighting function is  $w(p) : [0, 1] \rightarrow [0, 1]$  such that:*

- $w(p)$  is  $C^2$ ;
- $w' > 0$  for all  $p \in [0, 1]$ ;

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<sup>5</sup>A more general representation of the contract is  $t_s = (B + Ay, p; B, 1 - p)$  for some fixed-payment  $B \geq 0$ . This representation is more realistic inasmuch as it leaves some non-zero base-pay to the agent. Throughout the paper I stick to the normalization  $B = 0$ .

- $w(0) = 0$  and  $w(1) = 1$ ;
- There exists  $\tilde{p} \in [0, 1]$  such that  $w'' < 0$  if  $p \in [0, \tilde{p})$  and  $w'' > 0$  if  $p \in (\tilde{p}, 1]$ ;
- $\lim_{p \rightarrow 0^+} w' = \infty$  if  $\tilde{p} > 0$ ;
- $\lim_{p \rightarrow 1^-} w' = \infty$  if  $\tilde{p} < 1$ ;
- There exists  $\hat{p} \in (0, 1)$  such that  $w(\hat{p}) = \hat{p}$  if  $\tilde{p} \in (0, 1)$ .

According to Assumption 4,  $w(p)$  is an increasing and two-times continuously differentiable function that maps the unit interval onto itself. The probability weighting function contains *at least* two fixed-points: one at  $p = 0$  and another one at  $p = 1$ . Furthermore,  $w(p)$  can exhibit three different shapes: a concave shape if  $\tilde{p} = 1$ , a convex shape if  $\tilde{p} = 0$ , and an inverse-S shape, that is first concave and then convex, whenever  $\tilde{p} \in (0, 1)$ . The latter shape generates an additional interior fixed-point,  $\hat{p} \in (0, 1)$ . Finally, we assume that when the weighting function has either a concave or an inverse-S shape, near zero probabilities are infinitely overweighted. Instead, when the weighting function has either a convex or inverse-S shape, near one probabilities are infinitely overweighted. Such extreme sensitivity at near zero and near one probabilities is displayed by well-known proposals of probability weighting functions, such as those suggested by [Prelec \(1998\)](#), [Goldstein and Einhorn \(1987\)](#), and [Tversky and Kahneman \(1992\)](#).<sup>6 7</sup>

All in all, the rank-dependent expected utility of the agent when offered  $t_s$  is:

$$RDU(t_s) = w(p)b(Ay) - c(e). \quad (2)$$

When  $w(p) = p$ , RDU collapses to expected utility theory (EUT, from here onward). An agent with risk preferences characterized by EUT exhibits the following expected utility when working under  $t_s$ :

$$\mathbb{E}(U(t_s)) = pb(Ay) - c(e). \quad (3)$$

Another theory of risk that incorporates distortions of probabilities through probability weighting functions is Cumulative Prospect Theory (CPT, henceforth) ([Tversky and Kahneman, 1992](#)). CPT is a more descriptive version of RDU. An agent with CPT preferences also exhibits sensitivity to probabilities. However, the agent with CPT preferences displays relativistic

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<sup>6</sup>Well-known non-continuous probability weighting functions such as those proposed by [Chateauneuf et al. \(2007\)](#) and [Kahneman and Tversky \(1979\)](#) also exhibit extreme sensitivity near certainty and impossibility. In fact, the non-continuity of these weighting functions stems from the observation that subjects exhibit strong distortions of probability for events with extreme likelihoods.

<sup>7</sup>As noted by [Wakker \(2010\)](#), a weighting function with *cavecity*, that is first concave and then convex, does not necessarily ensure the existence of an interior point fixed-point. However, the assumptions  $\lim_{p \rightarrow 0^+} w'(p) = \infty$  if  $\tilde{p} > 0$  and  $\lim_{p \rightarrow 1^-} w'(p) = \infty$  if  $\tilde{p} < 1$  along with *cavecity* guarantee the existence of an interior fixed point  $\hat{p} \in (0, 1)$ .

perception of outcomes with respect to a *reference point*. This relativistic perception of outcomes entails that the CPT agent exhibits different risk attitudes for the domain of gains, that is all outcomes of the contract ranked above the reference point, and the domain of losses, all outcomes of the contract ranked below the reference point. Despite this difference, the incentives generated by the probability contract on agents with CPT preferences are qualitatively similar to those experienced agent with RDU preferences. In the interest of space, I relegate the formal description of CPT preferences and the analysis of the incentives produced by the proposed contracts on agents with CPT preferences to Appendix B.

## 2.1. Probabilistic risk attitudes and their decomposition

As mentioned before, characterizing the agent’s risk preferences with RDU introduces *probabilistic risk attitudes* (Wakker, 2001, Tversky and Wakker, 1995, Wakker, 1994). These attitudes are the mere influence of the agent’s sensitivity to probabilities on his risk attitudes. Probabilistic risk attitudes are relevant to the present analysis for two reasons. First, as it will be demonstrated later on, probability contracts have the advantage over piece-rates that they enable the principal to induce probabilistic risk attitudes that can enhance labor supply. In particular, when the principal implements probability contracts with a probability that induces strong probabilistic risk seeking, she can obtain higher output levels than those that would have been obtained with piece-rates, at no extra cost. Second, I investigate the specific attribute of probabilistic risk attitudes that guarantees this relevant result. To that end, I decompose probabilistic risk attitudes into two components that will be described and defined below, to, later on in the model, examine how each of them affects the agent’s motivation when he works under  $t_s$ .

The following decomposition of probabilistic risk attitudes is based on Wakker (2010). The first component of probabilistic risk attitudes captures *motivational* deviations from EUT stemming from pessimist or optimist attitudes toward risk. These two opposing factors affect probability evaluations because of the agent’s irrational belief that unfavorable outcomes, in the case of pessimism, or favorable outcomes, in the case of optimism, realize more often. Pessimism is represented with a convex weighting function, which assigns large weights to worst-ranked outcomes and small weights to best-ranked outcomes. This representation has been the convention in the early theoretical literature on rank-dependence (Yaari, 1987, Chew et al., 1987). Conversely, optimism is represented with a concave weighting function, which assigns large decision weights to best-ranked outcomes and small decision weights to worst-ranked outcomes. Figure 1, presents a graphical example of pessimism and optimism.

The second component of probabilistic risk attitudes is likelihood insensitivity (Abdellaoui et al., 2011, Wakker, 2001, Tversky and Wakker, 1995). This component captures the notion

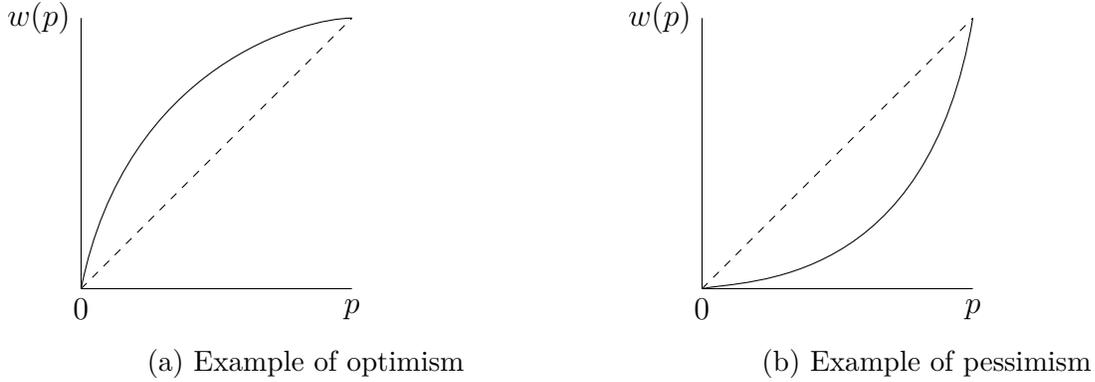


Figure 1: Motivational sources of probability distortion

that individuals distort probabilities because they are not sufficiently sensitive towards changes in intermediate probabilities but are overly sensitive to changes in extreme probabilities (Wakker, 2001). This deviation from EUT is due to cognitive and perceptual limitations. An extreme characterization of likelihood sensitivity is a probability weighting function  $w(p)$  with the properties of Assumption 4 plus the additional restrictions  $\hat{p} = 0.5$ ,  $\lim_{p \rightarrow 0^+} w(p) = 0.5$ , and  $\lim_{p \rightarrow 1^-} w(p) = 0.5$ . Such a weighting function entails that the agent accurately discriminates certainty and impossibility, but assigns  $p \approx 0.5$  to uncertain events. An opposing characterization to likelihood insensitivity is that of an agent who is fully sensitive to probabilities. Such agent's probability weighting function, ignoring the potential presence of optimism and pessimism, is  $w(p) = p$ . Figure 2 presents graphical examples of different degrees of likelihood insensitivity.

These two components of probability distortion are formally defined next:<sup>8</sup>

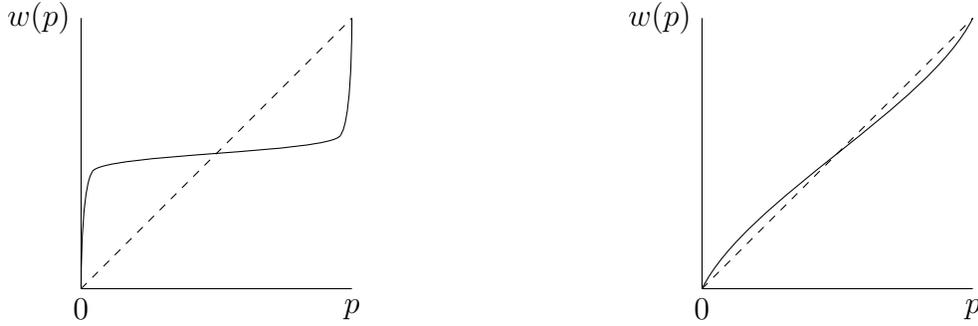
**Definition 1.** For a fully likelihood sensitive agent, pessimism (optimism) is represented by a probability weighting function  $w(p)$  with the properties of Assumption 4 and the additional restriction  $\tilde{p} = 0$  ( $\tilde{p} = 1$ ).

**Definition 2.** For an agent without optimism or pessimism, likelihood insensitivity is represented with a probability weighting function  $w(p)$  with the properties of Assumption 4 and the additional restriction  $\hat{p} = 0.5$ .

The co-existence of optimism or pessimism and likelihood insensitivity generates probabilistic risk attitudes that can be characterized by a probability weighting function with an inverse-S shape. The location of the interior fixed-point of such weighting function depends on whether the agent displays pessimism or optimism. A pessimist agent who is also likelihood insensitive,

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<sup>8</sup>These two phenomena have been addressed in the psychological literature as *curvature* and *elevation* (Gonzalez and Wu, 1999). I instead use the jargon used in economics. Pessimism was defined by Yaari (1987) and likelihood insensitivity was defined by Tversky and Wakker (1995).



(a) Example of extreme likelihood insensitivity (b) Example of moderate likelihood insensitivity

Figure 2: Cognitive sources of probability distortion

exhibits a  $w(p)$  with an interior fixed-point located in the interval  $\hat{p} \in (0, 0.5)$ . In contrast, an optimist agent who is also likelihood insensitive has a  $w(p)$  with an interior fixed-point in  $\hat{p} \in (0.5, 1)$ .<sup>9</sup>

When comparing the contracts  $t_s$  and  $t_d$ , special focus is given to the roles of likelihood insensitivity and optimism. These two components yield different requirements with regard to the implementation of the proposed contract. In particular, if likelihood insensitivity leads to higher output supply when  $t_s$  is offered, then the higher performance obtained with this type of contracts is due to cognitive limitations that can be inherent to the agent's perception of probability, and that can be readily available to the principal. Instead, if optimism yields that  $t_s$  generates higher output, then the principal needs to contract with agents that are optimistic when facing risk. This is a stringent requirement, given the abundant evidence that individuals are generally averse to risk, and thus pessimistic.

## 2.2. Contract comparisons

In this subsection, I compare the two considered contracts with respect to the output levels that they generate. To facilitate these comparisons, I make an assumption about the monetary incentives offered by both contracts. In particular, I assume that probability contracts offer, on expectation, the same monetary rewards as piece-rates. Formally, let  $A = \frac{a}{p}$ , so that  $\mathbb{E}(t_s) = ay = t_d$ . This equivalence allows me to focus on the incentives

<sup>9</sup>Additionally, the location of  $\hat{p}$  with respect to  $\tilde{p}$  in a probability weighting function is informative of the strength of optimism and pessimism with respect to likelihood insensitivity. When the convexity associated with pessimism outweighs the concavity generated by likelihood sensitivity in the interval  $p \in (0, 0.5)$ , the weighting function is convex at a larger interval of probabilities and  $\tilde{p} < \hat{p}$ . When the concavity associated with optimism outweighs the convexity generated by likelihood sensitivity in the interval  $p \in (0.5, 1)$ , the weighting function is concave at a larger interval of probabilities and  $\tilde{p} > \hat{p}$ . These conclusions are based on an additive representation of likelihood insensitivity and pessimism (optimism) in the weighting function, which is consistent with the independence of these two components (Wakker, 2010).

produced by probability contracts implemented with different probability  $p$  and how these incentives compare to those generated by piece-rates offering a similar monetary payment.<sup>10</sup>

Proposition 1 and Proposition 2 show that the effectiveness of the probability contract relative to the piece-rate depends on the agent's risk attitudes. When the agent's risk preferences are characterized by EUT, the probability contract generates higher output only when the agent is risk seeking, that is when the function  $b$  is convex. Therefore, under the standard property of diminishing marginal returns to money the proposed contract is counterproductive. Proposition 1 formally presents this result. The proofs of the main results of the paper are relegated to Appendix A.

**Proposition 1.** *Under Assumptions 1, 2 and 3, for an agent with any ability level and probability weighting  $w(p) = p$  the contract  $t_s$  generates higher output only if utility is strictly convex,  $b'' > 0$ .*

Proposition 1 formalizes the standard notion that introducing uncertainty in the agent's payment is counterproductive under the standard assumption of diminishing marginal returns to monetary rewards. Therefore, the principal would be better off incentivizing the agent with the piece-rate contract.

Instead, when the agent's preferences are characterized by RDU, risk attitudes do not only stem from the curvature of their utility function  $b$ , but also from the probabilistic risk attitudes generated by  $w$ . This codependence of utility curvature and probability weighting implies that, despite  $b$  being concave, the principal can choose a probability  $p$  that induces risk seeking attitudes in the agent and generate a preference for stochastic contracts. Such induced preference implies a boost in labor supply as compared to a setting in which the same agent were offered a cost-equivalent piece-rate. The following proposition formalizes this result:

**Proposition 2.** *Under Assumptions 1, 2, 3, and 4, for an agent with any ability level, concave utility  $b'' < 0$ , and probability weighting function  $w(p)$  displaying  $\hat{p} > 0$  and  $w''(p) > -\infty$ , there exists a probability  $p^* \in (0, \hat{p})$  such that the contract  $t_s$  generates higher output only if implemented with a probability satisfying  $p < p^*$ .*

A technical requirement included in Proposition 2 is that the agent's weighting function displays  $w''(p) > -\infty$ . Intuitively, the agent's diminishing marginal sensitivity to probability

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<sup>10</sup>A consequence of this assumption is that probability contracts nest piece-rates. Specifically, when the principal chooses to compensate output constantly, this is as  $p \rightarrow 1$ , then  $A \approx a$ . Conversely, in a setting in which the principal decides to evaluate output with little frequency, this is as  $p \rightarrow 0^+$ , the monetary incentives for delivering one additional unit of output become extreme in order to compensate the agent for the low frequency at which output is paid.

increments needs to be bounded. This requirement guarantees that whenever there exists a non-empty interval at which probabilities are overweighted, there must exist probabilities therein that, when included in the contract, induce risk seeking attitudes. A family of weighting functions that complies with this requirement is the constant relative sensitivity proposed by [Abdellaoui et al. \(2010\)](#) when their index of relative sensitivity is bounded. The axiomatic foundations of this family of weighting functions, their empirical validity, and empirical evidence of the boundedness of their curvature are also presented in [Abdellaoui et al. \(2010\)](#).

Next, I comment on the role of likelihood insensitivity and optimism/pessimism on the result presented in Proposition 2. Note that a crucial requirement in Proposition 2 is that  $w(p)$  exhibits  $\tilde{p} > 0$ , which ensures that probabilities are overweighted by the agent over some non-empty interval. Corollary 1 and Corollary 2 show that optimism and likelihood insensitivity are sufficient conditions for the existence of such an interval.

**Corollary 1.** *Optimism in the absence of likelihood insensitivity guarantees Proposition 2.*

**Corollary 2.** *Likelihood insensitivity in the presence of optimism or pessimism guarantees Proposition 2.*

To understand the intuition of Corollary 1 note that optimism is a sufficient condition to guarantee Proposition 2 since, absent likelihood insensitivity, it requires  $\tilde{p} = 1$  and, as a consequence, implies that the whole probability interval is overweighted by the agent. In contrast, when the agent exhibits pessimism, absent likelihood insensitivity, the entire probability interval is underweighted and Proposition 2 cannot hold. Therefore, agents who display optimism are more likely to be motivated by the incentives offered by the probability contract.

Moreover, Corollary 2 shows that likelihood insensitivity is a sufficient condition to guarantee Proposition 2. Likelihood insensitivity, absent pessimism or optimism, entails having a weighting function with a fixed-point  $\hat{p} = 0.5$  and, thus, that the interval of probabilities  $p \in (0, 0.5)$  is overweighted. When likelihood insensitivity coexists with optimism, the existence of an interval where probabilities are overweighted is trivial. Therefore, the most relevant implication of Corollary 2 is that likelihood insensitivity guarantees the existence of a non-empty interval where probabilities are overweighted despite the agent being pessimistic. The independence of pessimism and likelihood insensitivity, as proposed by [Wakker \(2010\)](#), and the continuity of  $w(p)$  ensure this result.

Corollary 2 has a relevant implication for the applicability of the proposed contract. In particular, it states that the principal does not necessarily need to contract with agents that are overly optimistic about the risk that is implied by these contracts to ensure their

effectiveness. Instead, cognitive factors that impede the accurate evaluation of probabilities guarantee the efficiency of these contracts. Therefore, likelihood insensitivity emerges as a deviation from standard preferences that can be targeted by the principal using the type of stochastic contracts proposed in this paper.

To summarize, Proposition 1 and Proposition 2 yield opposing results regarding the effectiveness of the proposed contract when individuals exhibit concave  $b$ . The theory used to characterize the agent’s risk preferences as well as the probability specified in the probability contract are crucial in generating this antagonism. An alternative analysis that not only incorporates the agent’s incentive compatibility, as done throughout this section, but that also considers the solution to the full principal’s program is presented in Appendix C. Such analysis confirms the main conclusion presented above: when contracting with an agent with RDU preferences, the principal is better off implementing probability contracts as long as they specify a probability that is severely overweighted by the agent. Moreover, Corollary 1 and Corollary 2 show that either optimism or likelihood insensitivity ensure the effectiveness of the probability contract proved in Proposition 2. The predictions derived from this theoretical framework are presented in Section 4, after the experimental design and procedures have been described.

### 3. Experimental Method

The experiment was conducted at Tilburg University’s CentERlab. The participants were all students at the university and were recruited using an electronic system. The data consist of 15 sessions with a total of 172 subjects. On average, a session lasted approximately 80 minutes. Between eight and eighteen subjects took part in a session. The currency used in the experiment was Euros. I used Z-Tree (Fischbacher, 2007) to implement and run the experiment. Subjects earned on average 15.83 Euros. The instructions of the experiment are presented in Appendix D.

The experiment consisted of two parts. Upon arrival, participants were informed that their earnings from either part one or those from part two would become their final earnings and that this would be decided by chance at the end of the experiment.<sup>11</sup> In the first part

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<sup>11</sup>This randomization of payments could be a source of concern if subjects distort probabilities. However, as explained later on in the text, the experiment was designed under the assumption of isolation (Tversky and Kahneman, 1981). This assumption entails that subjects evaluate and make decisions in each part of the experiment without considering the potential decisions to be made or the decisions made in the other part. As shown in Section 5 the assumption of isolation is corroborated by the data in the context of the real effort task. Moreover, as it will be shown in Section 6 it is found that on average subjects exhibit  $w(p) \approx 0.5$ , so, without the assumption of isolation, the probability underlying this randomization of payments was not

of the experiment subjects performed a task that demanded effort and attention. The task consisted of summing five two-digit numbers. Each summation featured randomly drawn numbers by the computer, ensuring similar levels of difficulty. When a participant knew the answer to the numbers that appeared in his screen, he could submit it using the computer interface. Immediately after submission, a new summation appeared on the computer screen and the participant was invited to solve it. In total, subjects had 10 rounds of four minutes each to complete as many summations as they could.

There were four treatments that differed in the incentives given to subjects to perform the task. Subjects were randomly assigned to one of these treatments. The baseline treatment is *Piecerate*. Subjects assigned to this treatment were paid 0.25 Euros for every correctly solved summation. The other three treatments also offered monetary rewards that depended on the individual performance of subjects on the task. However, in these treatments correct summations in some of the rounds, chosen at random at the end of the experiment, counted toward the subjects' earnings. These treatments were designed to represent probability contracts implemented with different probabilities. In particular, the treatments *LowPr*, *MePr* and *HiPr* featured a low, a medium, and a high probability, respectively, that performance in a round counted toward the subjects' earnings. In *LowPr* one round was randomly chosen at the end of the experiment and performance in that round was paid. Similarly, in *MePr* and *HiPr*, three and five rounds, respectively, were randomly chosen at the end of the experiment and performance in those rounds was paid. This experimental representation of the probability contract assumes that subjects exhibit isolation (Tversky and Kahneman, 1981), which in this setting implies that the subjects' decision to exert effort in a round does not take into account effort decisions made or to be made in other rounds. Under the condition of isolation, the treatments *LowPr*, *MePr*, and *HiPr*, generate uncertainty about whether the performance to be supplied in the round counts towards performance.<sup>12 13</sup>

As in the theoretical framework, the monetary incentives offered in *Piecerate*, *LowPr*, *MePr* and *HiPr* were calibrated such that subjects faced, on expectation, similar monetary incentives across the treatments. For instance, a subject assigned to *LowPr* received 2.50

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overweighted or underweighted.

<sup>12</sup>Isolation is strongly supported by the literature of experimental economics when the *random incentive system*, i.e. paying one round or one exercise at random at the end of the experiment, is implemented (See for instance Baltussen et al. (2012), Hey and Lee (2005) and Cubitt et al. (1998)).

<sup>13</sup>A common misunderstanding regarding the random incentive system is assuming that the independence axiom is a necessary condition to guarantee appropriate experimental measurement, which in the present setup implies subjects making effort choices as if each decision was paid *and* in the absence of income effects. While the independence axiom, along with some dynamic principles, is sufficient to guarantee proper measurement, this axiom not a necessary condition for experimental measurement. That is because isolation ensures proper experimental measurement under the random incentive system even if the independence axiom does not hold (Baltussen et al., 2012).

Euros for each correct summation in the round that was chosen for compensation, which was tenfold of what a subject assigned to Piecerate earned for each correctly solved summation. This difference in monetary payments exactly accounts for the difference in the probability that performance in a round is paid across the treatments. Similarly, subjects assigned the MePr and HiPr treatments received a compensation of 0.85 and 0.50 Euros, respectively, for each correctly solved summation in the rounds that were randomly chosen for compensation.

The probabilities governing the treatments LowPr, MePr and HiPr were chosen according to the common finding in the literature of decision-making: subjects distort probabilities according to an inverse-S shape probability weighting function with an interior fixed point at approximately  $p = 0.33$  (See [Wakker \(2010\)](#) pp.204 for a complete list of references finding this pattern). If subjects in the experiment follow this empirical regularity, they should on average overweight the probability that a round is chosen with 10% chance, underweight the probability that a round is chosen with 50% chance, and approximately evaluate accurately the probability that a round is chosen with 30% chance. Hence, the experiment was designed to observe performance differences across the treatments as long as the incentives generated by probability distortions are sufficiently strong.

Once the last round of the real-effort task was over, and before any feedback was given, participants were asked to state their beliefs about how well they did in the real-effort task. A subject received a bonus of one Euro if his answer was exactly equal to the number of correct summations that he performed over the ten rounds. This elicitation was unanticipated and the monetary compensation when the subject provided a correct answer was small as compared to the other sources of earnings in the experiment. These two characteristics ensure incentive compatibility ([Schlag et al., 2015](#)). The purpose of this belief elicitation is twofold. First, assess whether subjects anticipated the effect of the treatments on their own performance. Second, adapt these beliefs and use them as a reference point in some analyses of the data. Specifically, these beliefs about performance can be adapted to represent the subjects' belief about the amount of money they expect to earn in this part of the experiment.

In the second part of the experiment, the subjects' task was to choose between pairwise lotteries. This part of the experiment was designed to elicit their utility and the probability weighting functions. To elicit these two functions, I used the two-step method developed by [Abdellaoui \(2000\)](#). This method has the advantage of not making assumptions about the way in which subjects evaluate probabilities nor the way in which they evaluate monetary outcomes.<sup>14</sup>

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<sup>14</sup>A drawback of this method is that it is not incentive compatible when subjects are aware of the chained nature of the questions that they face. I overcome this disadvantage by randomly adding questions that are not used in the analysis of the data and by randomizing the appearance of the lotteries corresponding to decision sets 7 to 11.

This part of the experiment consisted of 11 decision sets. Decision sets 1 to 6 constitute the first step of Abdellaoui (2000)'s methodology, which is based on Wakker and Deneffe (1996). These decision sets elicit a sequence of outcomes  $\{x_1, x_2, x_3, x_4, x_5, x_6\}$  that made the subject indifferent between a lottery  $L = (x_{j-1}, 2/3; 0.5, 1/3)$  and a lottery  $R = (x_j, 2/3; 0, 1/3)$  for  $j = \{1, \dots, 6\}$ . Indifference was found through bisection. Specifically, a subject was required to express his preference between two initial lotteries. After having made a choice, the outcome  $x_j$  of lottery  $R$  changed as a function of the subject's choice, such that either the outcome of the chosen lottery was replaced by a less attractive alternative, or the outcome of the not chosen one was replaced by a more attractive alternative, while the other lottery remained the same. When facing the new situation, the subject was invited to make a choice again between the modified lotteries  $L$  and  $R$ . This process was repeated four times for each decision set. The left panel of Table 1 presents an example illustrating the bisection procedure used for Decision sets 1 to 6.

The lotteries in decision sets 1 to 6 were designed such that the resulting sequence  $\{x_1, x_2, x_3, x_4, x_5, x_6\}$  yielded equally spaced utility levels for each subject, i.e.  $u(x_j) - u(x_{j-1}) = u(x_{j-1}) - u(x_{j-2})$  for all for  $j = \{1, \dots, 6\}$ .<sup>15</sup> The starting point of the program,  $x_0$ , was set such that the monetary outcomes used in the lotteries reflected the subject's earnings in the first part of the experiment. In particular,  $x_0$  was set at  $\frac{2}{5}$ th of what a subject earned in the first part of the experiment. The advantage of using monetary outcomes of similar magnitude as the incentives offered in the real-effort task, is that I can more accurately relate the behavior of the subjects in such task with their elicited preferences. This is particularly relevant for the curvature of the utility function which can exhibit more curvature, and entail more risk aversion, if the incentives of this part of the experiment were substantially larger. Subjects were not informed about this calibration.

Decision sets 7 to 11 constitute the second step of Abdellaoui (2000)'s methodology. These decision sets were designed to elicit a sequence of probabilities,

$$\{w^{-1}(p_1), w^{-1}(p_2), w^{-1}(p_3), w^{-1}(p_4), w^{-1}(p_5)\},$$

where  $p_{j-1} = \frac{j-1}{6}$  and  $j = \{2, \dots, 6\}$ . These probabilities made the subjects indifferent between the lottery  $L = (x_6, w^{-1}(p_{j-1}); x_0, 1 - w^{-1}(p_{j-1}))$  and the degenerate lottery  $x_{j-1}$ . Note that these two lotteries were designed so that the elicited probabilities yield equally spaced probability weights, i.e.  $w(p_j) - w(p_{j-1}) = w(p_{j-1}) - w(p_{j-2})$  for  $p_{j-1} = \frac{j-1}{6}$  and  $j = \{2, \dots, 6\}$ . Again, indifference between these lotteries was found through bisection, with the probability of lottery  $L$  changing as a function of the subject's previous choices. The right

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<sup>15</sup>Note that indifference between  $L$  and  $R$  implies  $w(1/3)u(x_{j-1}) + (1 - w(1/3))u(0.5) = w(1/3)u(x_j) + (1 - w(1/3))u(0)$  which is equivalent to  $u(0.5) - u(0) = u(x_j) - u(x_{j-1})$  for any  $j = \{1, \dots, 6\}$

panel of Table 1 presents an example illustrating the bisection procedure for these decision sets.

Table 1: Example of the Abdellaoui’s (2000) algorithm

	<b>Left Panel</b>			<b>Right Panel</b>		
iteration #	Lotteries	Interval	Choice	Lotteries	Probability	Choice
1	L=(1, 0.66; 0.50, 0.33) R=(3.7, 0.66; 0, 0.33)	[1, 6.4 ]	L	L=( $x_1$ , 1) R=( $x_6$ , 0.50; 1, 0.5)	[0, 1]	L
2	L=(1, 0.66; 0.50, 0.33) R=(5.05, 0.66; 0, 0.33)	[3.7,6.4]	R	L=( $x_1$ , 1) R=( $x_6$ , 0.75; 1, 0.25)	[.5, 1]	L
3	L=(1, 0.66; .050, 0.33) R=(4.38, 0.66; 0, 0.33)	[3.7,5.05]	R	L=( $x_1$ , 1) R=( $x_6$ , 0.87; 1, 0.13)	[.75, 1]	R
4	L=(1, 0.66; 0.50, 0.33) R=(4.04, 0.66; 0, 0.33)	[3.7,4.38]	L	L=( $x_1$ , 1) R=( $x_6$ , 0.81; 1, 0.19)	[.75, .87]	L
5	L=(1, 0.66; 0.50, 0.33) R=(4.21, 0.66; 0, 0.33)	[4.04,4.38]	L	L=( $x_1$ , 1) R=( $x_6$ , 0.85; 1, 0.15)	[.81, .87]	L
		$x_1 \in [4.21, 4.38]$			$p_1 \in [0.85, 0.87]$	

Note: This table illustrates the bisection method used to elicit utility and probability functions. The lotteries in this table are expressed in the form  $(m, p; n, 1 - p)$  where  $m$  and  $n$  are prizes, and  $p$  is a probability. The left panel presents the bisection method to elicit utility and the right panel presents the bisection method to elicit probability functions.

Only after the the second part of the experiment was over, subjects were given feedback about their performance in the real-effort task, were told which round(s) counted toward payment if assigned to LowPr, MePr or HiPr, and were informed whether their belief was correct. Also, they were informed about the lottery that was chosen for compensation for the second part of the experiment and its realization. In addition, subjects learned whether part one or part two counted toward their final earnings.

## 4. Hypotheses

The theoretical model generates a set of hypotheses that will be tested with the experiment. The first hypothesis is based on Prediction 2, which demonstrates that probability contracts can outperform piece-rates despite utility being concave. To obtain such result, the probability contract needs to be implemented with a probability that is sufficiently overweighted by the agent, which induces a taste for stochastic contracts and, thus, a boost in labor supply when the agent is incentivized with the probability contract as compared to the hypothetical case in which he were offered the piece-rate.

As stated in Section 3, if subjects in the experiment conform to the common finding that their probability weighting function exhibits a fixed-point at  $\hat{p} \approx 0.33$  and the incentives

created by the probability contract are strong, then subjects assigned to LowPr should display higher performance than subjects in Piecerate. Instead, subjects in HiPr should display lower performance than those in Piecerate, and the performance levels between subjects in Piecerate and MePr should be indistinguishable. These conjectures are included in Hypothesis 1.<sup>16</sup>

**Hypothesis 1.** *Subjects with risk preferences characterized by RDU and a weighting function  $w(p)$  with an interior fixed-point at  $\hat{p} \approx 0.33$  exhibit average performance levels that conform to the ranking:*

$$LowPr > MePr = Piecerate > HiPr.$$

Empirical support in favor of Hypothesis 1 does not conclusively validate the proposed model. It is possible that factors other than probabilistic risk attitudes could spawn these performance differences. Hence, if the model is an adequate representation of behavior and incentives, performance differences between the LowPr and Piecerate and between the HiPr and Piecerate should be explained by the subjects' tendency to overweight small probabilities and underweight large probabilities, respectively.

**Hypothesis 2.** *Subjects assigned to LowPr (HiPr) who have a weighting function that overweights (underweights) small (large) probabilities exhibit higher (lower) performance with respect to subjects assigned to Piecerate.*

Finally, if Hypothesis 1 and Hypothesis 2 are corroborated by the experimental data, I am interested in understanding the component of probabilistic risk attitudes that causes the differences between the treatments. Corollary 1 predicts that optimistic agents, independently of whether they are likelihood insensitive or not, should display higher performance when assigned to the treatments representing probability contracts.

**Hypothesis 3.** *Optimistic subjects exhibit higher performance when assigned to the treatments representing the probability contract as compared to subjects in Piecerate.*

Furthermore, Corollary 2 predicts that likelihood insensitive subjects, regardless of whether they also exhibit optimism or pessimism, are more likely to display performance differences across the treatments according to the ranking predicted by Hypothesis 1.

**Hypothesis 4.** *Likelihood insensitive subjects are more likely to exhibit performance differences according to the ranking shown in Hypothesis 1.*

The accuracy of these hypotheses will be evaluated with the experimental data in the next section.

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<sup>16</sup>Note that 1 contradicts the standard notion that stochastic contracts are counterproductive since they induce uncertainty in the agent's payment, formalized in Proposition 1.

## 5. Results

### 5.1. Treatment effects

In this subsection I compare performance in the effort task across the treatments. Performance is defined as the total number of correctly solved summations by a subject. Table 2 presents the descriptive statistics of performance by treatment. This table shows that, as predicted by Hypothesis 1, the probability contract with  $p = 0.10$  generates higher performance than the piece-rate contract. Specifically, subjects assigned to the LowPr treatment solved on average 20.56 % more summations than subjects assigned to Piecerate ( $t(84.454) = 2.361, p = 0.010$ ).<sup>17</sup> The effect size of this difference in performance is of 0.5 standard deviations which is significant at the 5% confidence level.<sup>18</sup>

Table 2: Descriptive statistics of performance by treatments

Treatment	LowPr	MePr	HiPr	Piecerate	Total
Mean	98.116	87.900	83.75	81.377	87.686
Median	91	87	82.500	77	85
St.dev.	34.659	28.134	24.358	31.684	30.412
N	43	40	44	45	172

In contrast, probability contracts implemented with higher probabilities generate similar average performance as the piece-rate contract. Subjects assigned the MePr treatment solved 87.9 correct summations on average and subjects assigned the HiPr treatment solved 83.7 correct summations on average, neither of which are statistically different from the average number of correct summations solved by subjects assigned to Piecerate.<sup>19</sup> These findings partially support Hypothesis 1, which accurately predicts that MePr induces similar performance as Piecerate, but incorrectly predicts that HiPr generates lower performance than Piecerate. Conjectures about this partial confirmation of Hypothesis 1 are provided at the end of the subsection.

Among the treatments representing the probability contract, the LowPr generates greater average performance. This treatment generates 17% higher average performance than

<sup>17</sup>A Wilcoxon-Mann-Whitney test also rejects the null hypothesis of no performance difference between Piecerate and LowPr ( $z = 2.634, p < 0.01$ )

<sup>18</sup>The significance of the effect size was evaluated with a bootstrapped 95% confidence interval with 10000 repetitions.

<sup>19</sup>The t-tests of these comparisons are ( $t(83) = 1.005, p = 0.159$ ) and ( $t(82.44) = -0.386, p = 0.692$ ), respectively. Wilcoxon-Mann-Whitney tests of these comparisons yield ( $z = 1.321, p = 0.186$ ) and ( $z = -0.895, p = 0.3710$ ), respectively.

HiPr ( $t(75.215) = 2.232, p = 0.014$ ), and 11% higher average performance than MePr ( $t(79.575) = 1.478, p = 0.0716$ ).<sup>20</sup> Therefore, statistical inference using pairwise testing suggests that LowPr generates highest average performance, while the other treatments produce similar performance.

I estimate regressions of each subject’s performance on treatment dummies, dummies that capture different shapes of the utility function as well as dummies that capture different shapes of the weighting function. These regressions have the purpose of establishing the robustness of the aforementioned treatment effects when the average risk attitude of subjects is controlled for. That these treatment effects are robust to the inclusion of these controls indicates that the performance differences between the treatments are not an artifact of more risk seeking or less risk averse subjects assigned to some of the treatments. The dummies reflected individual classifications of utility and weighting functions based on the subject’s answers to the second part of the experiment. On the one hand, a utility function can be classified as having linear, concave, convex, or mixed shape. Details of this classification are provided in Appendix F.<sup>21</sup> On the other hand, a probability weighting function can be classified as displaying lower subadditivity (LS, from here onward) and/or upper subadditivity (US, from here onward). A weighting function with LS assigns larger decision weights to best-ranked outcomes than to middle-ranked outcomes. A weighting function with US assigns larger decision weights to worst-ranked outcomes than middle-ranked outcomes.<sup>22</sup> An alternative classification for weighting functions focuses on the strength of the possibility effect relative to the certainty effect. The variable “Possibility” takes a value of one if the possibility effect is stronger than the certainty effect and zero otherwise.<sup>23</sup> Details of these classifications are provided in Appendix G.

Table 3 presents the regression estimates. For all specifications, the coefficient associated to assignment to LowPr is significant and positive at the 5% significance level, which corroborates the aforementioned result that subjects assigned to that treatment display higher average performance than subjects assigned to Piecerate, the benchmark treatment of the regression. Similarly, the coefficient of LowPr is significantly higher than the estimate associated with HiPr ( $F(1, 159) = 6.58$ ) and significantly higher than that associated to

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<sup>20</sup>Wilcoxon-Mann-Whitney tests of these differences yield ( $z = 1.96, p = 0.049$ ) and ( $z = 1.035, p = 0.07$ ), respectively. In addition, the effect sizes of these differences are of 0.4805 standard deviations and 0.322 standard deviations, respectively. Both of which are significant at the 10 % level.

<sup>21</sup>In short, a variable  $\Delta_j'' := (x_j - x_{j-1}) - (x_{j-1} - x_{j-2})$  for  $j = 2, 3, 4, 5, 6$ , is constructed for each subject. A subject is classified as having linear utility if most values  $\Delta_j''$  are close to zero, concave utility if most values  $\Delta_j''$  are positive, convex utility if most values  $\Delta_j''$  are negative, and mixed utility otherwise.

<sup>22</sup>In short, a subject in the experiment exhibits LS when  $w^{-1}(\frac{1}{6}) < w^{-1}(\frac{2}{6}) - w^{-1}(\frac{1}{6})$ . A subject exhibits US when  $1 - w^{-1}(\frac{5}{6}) < w^{-1}(\frac{5}{6}) - w^{-1}(\frac{4}{6})$ .

<sup>23</sup>A subject has a possibility effect that is stronger than the certainty effect when  $1 - w^{-1}(\frac{5}{6}) < w^{-1}(\frac{1}{6})$ .

MePr ( $F(1, 159) = 6.02$ ). Thus, among the studied contracts, the LowPr produces the highest performance and the performance generated by the other contracts is statistically indistinguishable.

Table 3: Regression of performance on treatments

	(1)	(2)	(3)	(4)
	Performance	Performance	Performance	Performance
LowPr	16.739** (7.090)	16.558** (7.508)	16.001** (7.532)	16.526** (7.589)
MePr	6.522 (6.487)	6.714 (6.610)	6.335 (6.677)	6.585 (6.724)
HiPr	2.372 (5.985)	1.684 (5.888)	1.616 (6.308)	0.758 (6.016)
Concave		14.359 (9.401)	15.067 (9.529)	15.090 (9.681)
Convex		7.623 (10.109)	8.527 (10.469)	7.185 (10.513)
Mixed		3.864 (6.625)	3.698 (6.699)	4.259 (6.785)
US			0.904 (5.183)	
LS			2.924 (5.053)	
Possibility				4.901 (7.637)
Certainty				7.062 (7.791)
Constant	81.378*** (4.726)	79.819*** (5.025)	78.497*** (5.242)	74.667*** (7.371)
R <sup>2</sup>	0.045	0.062	0.065	0.064
Observations	172	172	172	172

Note: This table presents the estimates of the Ordinary Least Squares regression of the model  $Performance_i = \gamma_0 + \gamma_1 LowPr + \gamma_2 MePr + \gamma_3 HiPr + Controls' \Lambda + \epsilon_i$ , with  $E(\epsilon_i | MePr, LowPr, HiPr, Controls) = 0$ . “Performance” is the number of correctly solved sums solved by a subject in the first part of the experiment, “Piecerate”, “LowPr”, “MePr” and “HiPr” are binary variables that indicate if a subject was assigned to the treatment offering a piece-rate or probability contracts implemented with low, medium or high probability, respectively. “Piecerate” is the benchmark of the regression. Robust standard errors in parenthesis. \*\*\* denotes significance at the 0.01 level, \*\* denotes significance at the 0.05 level, \* denotes significance at the 0.1 level.

A possible explanation to these results is that LowPr generates higher performance because it circumvents income effects (See [Azrieli et al. \(2018\)](#) and [Lee \(2008\)](#)). In contrast, these

effects are present in Piecerate and they can be a source of demotivation for subjects assigned to this treatment. This rationale also accommodates the result that LowPr generates higher performance than MePr and HiPr because, by paying less rounds of the real-effort task, it is more effective in minimizing income effects. However, the data show that significant performance differences across the treatments emerge in the first round, when income effects are absent. A regression of performance in a given round on treatment dummies, round dummies, and relevant controls is estimated with standard errors clustered at the individual level. The estimates of this regression show that in the first round subjects assigned to LowPr achieve 1.6 higher average summations as compared to subjects in Piecerate ( $p = 0.019$ ).<sup>24</sup>

While I find compelling evidence of performance differences across some of the treatments, the data on subjects' beliefs suggest that they are statistically indistinguishable across the treatments. This finding suggests that subjects in the experiment did not anticipate the motivational effect of the probability contract implemented with probability  $p = 0.10$ . The fact that individuals do not anticipate the motivational effects of the proposed contracts, can explain why these contracting modalities are scarcely observed in practice. Appendix E presents a detailed analysis of these data and a broader discussion of the implications of this result.

All in all, the data on performance in the real-effort task *partly* supports Hypothesis 1. However, recall that Hypothesis 1 was structured around the common finding that individuals overweight all probabilities up to  $p = 0.33$  and underweight all probabilities thereafter. Instead, the analyses presented in this subsection suggested that subjects in the experiment overweighted on average the probability  $p = 0.10$  and evaluated approximately accurately the probabilities  $p = 0.3$  and  $p = 0.5$ . In the next subsection, I show that subjects indeed display an average probability weighting function with such a shape.

## 5.2. Probability weighting functions

In this subsection, I analyze the data of the second part of the experiment. These data feature the subjects' choices between pairwise lotteries, which were designed to elicit the subjects' utility and probability weighting functions. The analysis of these data show that subjects display an average probability weighting function with a strong inverse-S shape and with more optimism than that documented in previous studies. This shape induces a strong overweighting of small probabilities as well as moderate underweighting of medium to large probabilities.

As explained in Section 3, the second part of the experiment consisted of 11 decision sets.

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<sup>24</sup>Subjects in LowPr also exhibit higher average performance as compared to subjects in MePr ( $F(1, 171) = 2.21, p = 0.069$ , one tailed ) and subjects in HiPr ( $F(1, 171) = 5.04, p = 0.026$ )

Decision sets 1 to 6 were designed to elicit the sequence  $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ , representing the subjects' preferences over monetary consequences. Different analyses of the data demonstrate that the majority of subjects exhibit linear utility functions over the monetary outcomes of the lotteries, which is in line with the findings of [Wakker and Deneffe \(1996\)](#) and [Abdellaoui \(2000\)](#), as well as with the critique put forward by [Rabin \(2000\)](#). Given this result and since the main focus of the paper is on probability weighting functions and their influence on the effectiveness of the proposed contract, I relegate the complete analysis of these data to Appendix F.

Decision sets 7 to 11 of the second part of the experiment were designed to elicit the sequence

$$\{w^{-1}(p_1), w^{-1}(p_2), w^{-1}(p_3), w^{-1}(p_4), w^{-1}(p_5)\},$$

with  $p_{j-1} = \frac{j-1}{6}$  and  $j = \{2, \dots, 6\}$ . These data are analyzed to examine how subjects evaluated probabilities. To that end, I perform regressions at the individual level that relate the elicited probabilities to the probability weights that they map. The rationale for using regressions as the main analysis of these data is that i) they provide a good indication of the average degree of probability distortion in the experiment, ii) the resulting estimates can be used to compare the degree of probability distortion in the experiment to those reported in previous studies, and iii) with the resulting estimates one can construct indexes of likelihood insensitivity and optimism, which according to [Corollary 1](#) and [Corollary 2](#) are critical to understand the determinants behind the efficiency of the probability contracts.<sup>25</sup> Alternative analyses of these data, including non-parametric analyses performed at the individual level, are presented in Appendix G. I perform the regressions assuming different and well-known functionals of probability weighting. Specifically, I use the neo-additive probability weighting function ([Chateauneuf et al., 2007](#)), [Tversky and Kahneman \(1992\)](#) probability weighting function, [Prelec \(1998\)](#) two-parameter probability weighting function, and [Goldstein and Einhorn \(1987\)](#) log-odds probability weighting function. Performing the regressions with different parametric functions ensures robustness, i.e. the results do not stem from the underlying assumptions of a particular functional form.

[Table 4](#) presents the regression estimates. Panel 1 presents the estimates of a truncated regression of the neo-additive functional,  $w(p) = c + sp$ .<sup>26</sup> The resulting estimates display  $\hat{c} > 0$  and  $\hat{c} + \hat{s} < 1$ , which imply that subjects on average overweighted small probabilities and underweighted large probabilities. Furthermore,  $\hat{c}$  and  $\hat{s}$  are larger and smaller, respectively,

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<sup>25</sup>Comparisons across studies must be taken with a grain of salt inasmuch as resulting differences cannot only be attributed to differences in preferences, but also to the different stakes and methods used to elicit risk preferences.

<sup>26</sup>The assumed truncation at the extremes,  $w(0)$  and  $w(1)$ , provides the estimation with the flexibility to admit weighting functions with S-shape.

than the estimates reported in [Abdellaoui et al. \(2011\)](#), suggesting that subjects in my experiment exhibit higher degrees of optimism toward risk as well as higher degrees of likelihood insensitivity.

A more traditional parametric representation of the probability weighting function was proposed by [Tversky and Kahneman \(1992\)](#). Their proposal relates probabilities and their associated weights according to the following non-linear function:  $w(p) = \frac{p^\psi}{(p^\psi + (1-p)^\psi)^{\frac{1}{\psi}}}$ . The second panel of [Table 4](#) shows that the non-linear least squares method generates an estimate  $\hat{\psi} = 0.59$ , which is lower than those reported in previous studies. Specifically, classical experiments report estimates in the range 0.60-0.75 ([Bleichrodt and Pinto, 2000](#), [Abdellaoui, 2000](#), [Wu and Gonzalez, 1996](#), [Tversky and Kahneman, 1992](#)). Therefore, subjects in my experiment display a weighting function with more severe probability distortion.

A crucial disadvantage of Tversky and Kahneman’s (1992) weighting function is that likelihood insensitivity and optimism/pessimism influence  $\psi$ , so their effect on probabilistic risk attitudes cannot be identified. To overcome such disadvantage, I also use the log-odds weighting function proposed by [Goldstein and Einhorn \(1987\)](#),  $w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$ , which can, up to some extent, separate these two components. The estimates of a non-linear least squares regression are presented in [Panel 3](#). The magnitude of  $\hat{g}$  indicates that the average weighting function has a strong inverse-S shape and the magnitude of  $\hat{\delta}$  a strikingly small degree of pessimism. These coefficients are lower and higher, respectively, than those found in previous studies ([Bruhin et al., 2010](#), [Bleichrodt and Pinto, 2000](#), [Abdellaoui, 2000](#), [Gonzalez and Wu, 1999](#), [Wu and Gonzalez, 1996](#), [Tversky and Fox, 1995](#)). Thus, subjects in the experiment had an average weighting function with more likelihood insensitivity and optimism than previously documented.

Lastly, I also estimate a regression assuming [Prelec \(1998\)](#)’s probability weighting function with two parameters,  $w(p) = \exp(-\beta(-\ln(p))^\alpha)$ . This parametric functional also separates, up to some extent, optimism from likelihood insensitivity. [Panel 4](#) presents the estimates of a non-linear least squares regression. The estimate  $\hat{\alpha}$ , which is statistically lower than one, entails that the average probability function has a strong inverse-S shape. Moreover, the estimate  $\hat{\beta}$ , which is also statistically lower than one, entails that subjects on average display optimism. Previous estimations of this probability weighting function report larger values of  $\alpha$  and  $\beta$  ([Murphy and Ten Brincke, 2018](#), [Haridon et al., 2018](#), [Fehr-duda, 2012](#), [Abdellaoui et al., 2011](#), [Bleichrodt and Pinto, 2000](#)). Hence, these subjects display an average probability weighting function with a stronger inverse-S shape and more optimism as compared to previous studies.

All in all these estimations lead to the conclusion that subjects display an average probability weighting function with a strong inverse-S shape and more optimism than

Table 4: Parametric estimates of the weighting function

<b>Panel 1</b> $w(p) = c + sp$		
	$\hat{c}$	$\hat{s}$
	0.194*** (0.021)	0.566*** (0.036)
Log-Likelihood		220.288
N		860
<b>Panel 2</b> $w(p) = \frac{p^\psi}{(p^\psi + (1-p)^\psi)^{\frac{1}{\psi}}}$		
		$\hat{\psi}$
		0.598*** (0.016)
Adj. R <sup>2</sup>		0.838
N		860
<b>Panel 3</b> $w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$		
	$\hat{\gamma}$	$\hat{\delta}$
	0.281*** (0.025)	0.921*** (0.020)
Adj. R <sup>2</sup>		0.863
N		860
<b>Panel 4</b> $w(p) = \exp(-\beta(-\ln(p))^\alpha)$		
	$\hat{\alpha}$	$\hat{\beta}$
	0.284*** (0.025)	0.841*** (0.015)
Adj. R <sup>2</sup>		0.864
N		860

Note: This table presents estimates of the average probability weighting function of subjects when different parametric forms are assumed. Panel 1 presents the maximum likelihood estimates of the equation  $w(p) = c + sp$  when truncation at  $w(p) = 0$  and at  $w(p) = 1$  is assumed. Panel 2 presents the non-linear least squares estimation of the function  $w(p) = \frac{p^\psi}{(p^\psi + (1-p)^\psi)^{\frac{1}{\psi}}}$ . The third panel presents the non-linear least squares estimates of the parametric form  $\frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$ . The last panel presents the non-linear least squares estimates of the function  $w(p) = \exp(-\beta(-\ln(p))^\alpha)$ . Robust standard errors in parenthesis. \*\*\* denotes significance at the 0.01 level, \*\* denotes significance at the 0.05 level, \* denotes significance at the 0.1 level.

previously documented. The coexistence of these properties produces pattern of probability distortion whereby small probabilities are strongly overweighted and medium to large probabilities are moderately distorted. For instance, using the estimates of Panel 1 it can be established that for the subjects in this experiment the probability  $p = 0.10$  is on average perceived to be  $w(0.10) = 0.25$ , while the probabilities  $p = 0.30$  and  $p = 0.5$  are on average perceived to be  $w(0.30) = 0.363$  and  $w(0.50) = 0.477$ , respectively. These patterns of probability distortion accommodates the findings of the first part of the experiment, namely that LowPr generates higher output than Piecerate, and that HiPr, MePr, and Piecerate produce similar performance. In the next section, I conclusively demonstrate that the shape of the probability weighting functions of subjects explains the treatment effects.

For the sake of robustness, I perform the aforementioned estimations accounting for the possibility that subjects have CPT preferences. In such analysis I assumed the subject’s reference point to be the monetary equivalent of each subject’s belief in the first part of the experiment. Lottery outcomes above this reference point belong to the domain of gains, while lottery outcomes below this reference point belong to the domain of losses. I perform separate regressions for each domain. Estimates of these regressions are presented in Table 15 in Appendix H. I find that for all considered functional forms of probability weighting and for both domains, subjects display weighting functions with inverse-S shapes and more optimism than previously found. As a consequence, the results presented in this section are robust to the assumption that subjects’ preferences can be represented by CPT preferences.

### 5.3. Likelihood insensitivity and the treatment effect

This subsection reconciles the results of the two parts of the experiment. First, I present empirical evidence supporting Hypothesis 2. That is, I demonstrate that the higher average performance of subjects assigned to LowPr is caused by their tendency to overweight small probabilities. Second, I show that likelihood insensitivity, alone, explains the treatment effects documented in Section 5, validating Hypothesis 4.

To empirically verify the validity of Hypothesis 2, I extend the statistical models presented in Table 3 by including interactions between the variable indicating assignment to LowPr, the only treatment for which there is evidence of performance differences against the baseline treatment, and variables that capture the shape of the probability weighting function of a subject. Due to the nature of the treatment LowPr, I focus on variables that indicate whether a subject exhibits a weighting function with overweighting of small probabilities. Specifically, I use the variables LS, Possibility, and Overweight $_{p=\frac{1}{6}}$ . The first two variables were already defined in Section 5.1. The last variable takes a value of one if a subject overweightes the probability  $p = \frac{1}{6}$  and zero otherwise. These variables relate in the following way: a subject

for whom LS takes a value of one, overweights the probability  $p = \frac{1}{6}$  and might exhibit a possibility effect that is stronger than the certainty effect. Similarly, a subject for whom Possibility takes a value of one overweights  $p = \frac{1}{6}$  and is likely to exhibit LS.

I first describe the results of the analysis when the variable LS is used. Column (1) in Table 5 presents the OLS estimates of the extended regression. I find that subjects assigned to LowPr who have weighting functions with lower subadditivity display an average performance level that is significantly higher than that of subjects in Piecerate. In contrast, subjects assigned to LowPr with weighting functions without lower subadditivity display an average performance level that is statistically indistinguishable to that of subjects in Piecerate. Thus, only subjects with a weighting function assigning larger decision weights to small probabilities relative to the weights assigned to medium-ranged probabilities display higher performance levels when assigned to LowPr and, as a result, exhibit pronounced treatment effects.

The above conclusion is robust to using other variables that capture overweighting small probabilities. The OLS estimates in column (2) of of Table 5 show that subjects working under LowPr who overweighted the probability  $p = \frac{1}{6}$  exhibit higher average performance than subjects in Piecerate. Moreover, these subjects also exhibit a steeper treatment effect than that of subjects who were assigned to LowPr but who did not overweight the probability  $p = \frac{1}{6}$ . Moreover, the estimates in column (3) demonstrate that subjects assigned to LowPr and for who the variable Possibility takes a value of one, display a significant treatment effect. Instead, subjects assigned to LowPr and who do not exhibit a strong possibility effect do not display performance differences as compared to subjects in Piecerate.<sup>27</sup>

Thus, the results presented in Table 5 provide empirical evidence that subjects with weighting functions that induce overweighting of small probabilities display pronounced treatment effects. These findings are in line with Hypothesis 2.

We are now in a position to investigate the influence of likelihood insensitivity and optimism on performance. To that end, I first classify subjects according to their degree of likelihood sensitivity and optimism. Following Wakker (2010) and Abdellaoui et al. (2011), I estimate for each subject,  $i$ , the following neo-additive functional:

$$w(p_{ij}) = c_i + s_i p_{ij} + \epsilon_i,$$

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<sup>27</sup>Note that unlike the analyses in which LS and Possibility were assumed to be the underlying mechanism behind the treatment effects, the coefficient associated to LowPr remains significant when  $\text{Overweight}_{p=\frac{1}{6}}$  is assumed. This significance suggests that the treatment effect is not entirely captured by the mere tendency of subjects to overweight the probability  $p = \frac{1}{6}$ , but instead is captured by the tendency of subjects to overweight the probability  $p = \frac{1}{6}$  relative to other elicited probabilities, such as  $p = \frac{1}{2}$  and  $p = \frac{5}{6}$  which is what the variables LS and Possibility captures. This is a first suggestion of the empirical result that the overweighting of probabilities emerging from likelihood insensitivity, which entails such relative overweighting of small probabilities with respect to medium-sized probabilities, explains the treatment effect.

Table 5: The influence of probability overweighting on the treatment effects

	(1)	(2)	(3)
	Performance	Performance	Performance
LowPr*Mechanism	29.055** (12.056)	17.418* (10.302)	21.821** (9.822)
Mechanism	1.834 (7.380)	3.031 (6.005)	0.089 (7.394)
LowPr	7.248 (7.538)	17.459** (8.601)	2.745 (7.848)
MePr	6.582 (6.748)	6.543 (6.577)	6.852 (6.760)
HiPr	1.320 (6.235)	2.067 (5.985)	0.330 (5.967)
US	4.654 (6.275)		
BOTH	-9.950 (10.072)		
Certainty			6.233 (7.222)
Concave	16.373* (9.225)	14.570 (9.460)	15.431 (9.735)
Convex	10.449 (12.795)	7.656 (9.999)	8.525 (8.363)
Mixed	3.740 (6.854)	4.064 (6.749)	4.465 (6.851)
Constant	79.035*** (5.115)	78.899*** (5.471)	77.935*** (6.793)
Mechanism variable	LS	Overweight <sub><math>p=\frac{1}{6}</math></sub>	Possibility
R <sup>2</sup>	0.089	0.063	0.08
Observations	172	172	172

Note: This table presents the estimates of the Ordinary Least Squares regression of the model  $Performance_i = \gamma_0 + \gamma_1 LowPr + \gamma_2 LowPr * Mechanism + \gamma_3 Mechanism + \gamma_4 MePr + \gamma_5 MePr + \gamma_6 HiPr + Controls' \Gamma + \epsilon_i$ , with  $E(\epsilon_i | MePr, LowPr, HiPr, Controls, Mechanism) = 0$ . "Performance" is the number of correctly solved sums solved by a subject in the first part of the experiment, "Piecerate", "LowPr", "MePr" and "HiPr" are binary variables that indicate if a subject was assigned to the treatment offering a piece-rate or probability contracts implemented with low, medium or high probability, respectively. "Piecerate" is the benchmark of the regression. In column (1) Mechanism is equal to "LS" a binary variable that takes a value of one if a subject has a weighting function with lower subadditivity and zero otherwise. In column (2) Mechanism is equal to "Overweight <sub>$p=\frac{1}{6}$</sub> " a binary variable that takes a value of one if a subject overweights the probability  $p = \frac{1}{6}$ . In column (3) Mechanism is equal to "Possibility" a binary variable takes a value of one if a subject has a weighting function with the possibility effect being stronger than the certainty effect and zero otherwise. Robust standard errors in parenthesis. \*\*\* denotes significance at the 0.01 level, \*\* denotes significance at the 0.05 level, \* denotes significance at the 0.1 level.

where  $j$  indicates an elicited probability. The magnitude of the estimate  $\hat{s}_i$  indicates subject's  $i$  sensitivity to probabilities. If  $\hat{s}_i < 1$ , the subject is not sufficiently responsive to changes in probabilities and is classified as likelihood insensitive. Instead, if  $\hat{s}_i > 1$ , the subject is too sensitive to changes in probabilities and is classified as likelihood sensitive. I find that 102 subjects in my sample are likelihood insensitive and 61 subjects are classified as likelihood sensitive.<sup>28</sup> Importantly, the degree of likelihood insensitivity and that of likelihood sensitivity are balanced across treatments.

In addition, the magnitude of  $\hat{c}_i$  and that of the sum  $\hat{c}_i + \hat{s}_i$  determine the degree of optimism of subject  $i$ . Whenever  $\hat{c}_i > 0$  and  $\hat{c}_i + \hat{s}_i \leq 1$ , the subject assigns large weights to best-ranked outcomes and small decision weights to worst-ranked outcomes, and, as a consequence, exhibits optimism. Alternatively, if  $\hat{c}_i < 0$  and  $\hat{c}_i - \hat{s}_i \leq 1$ , the subject exhibits pessimism. I find that 80 subjects in my sample display optimism while 32 subjects display pessimism.<sup>29</sup> Again, the degrees of optimism are balanced across treatments.

Binary variables capturing the above classifications are added to the regressions presented in Table 3. Also, interactions between assignment to LowPr and the variables indicating whether a subject is likelihood insensitive and whether a subject exhibits optimism are included in these regressions. These interactions allow me to evaluate the strength of the treatment effect among likelihood insensitive subjects as well as the strength of the treatment effect among optimistic subjects.

The resulting regression estimates are presented in columns (1) and (2) of Table 6. All in all, I find empirical support for Hypothesis 4. Specifically, I find that likelihood insensitive subjects assigned to LowPr display higher average performance as compared to subjects assigned to Piecerate. In contrast, subjects assigned to LowPr and who were not classified as likelihood insensitive did not exhibit performance differences with respect to the baseline treatment. These findings support the theoretical result that likelihood insensitivity ensures the efficiency of probability contracts when these are implemented with a small probability. In addition, the data show that subjects displaying optimism and who were assigned to LowPr exhibit average performance levels that are statistically indistinguishable from those of subjects in Piecerate. Therefore, optimism, alone, is unable to explain the treatment effects documented in Section 5.1. This result invalidates Hypothesis 3.

For the sake of robustness, I also estimate for each subject the parameters of the probability

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<sup>28</sup>Nine subjects had a negative estimated parameter  $\hat{s}_i$  which has no clear interpretation and are thus left unclassified.

<sup>29</sup>60 subjects are left unclassified since they display  $\hat{c}_i + \hat{s}_i > 1$ , which has no clear interpretation. In an alternative classification that ignores the restriction  $\hat{c}_i + \hat{s}_i \leq 1$ , these subjects are classified to be either optimistic or pessimistic. Under this alternative classification the relevant result that likelihood insensitivity explains the treatment effects, presented later on, also emerges.

Table 6: The influence of likelihood insensitivity and optimism on the treatment effects

	(1)	(2)	(3)	(4)
	Performance	Performance	Performance	Performance
LowPr*Likelihood ins.		24.182** (10.625)	27.209** (12.607)	23.654** (11.947)
LowPr*Optimism		-3.112 (9.958)	17.081 (16.404)	-3.594 (32.031)
LowPr	15.636** (6.522)	6.812 (11.115)	10.249 (13.613)	5.154 (11.850)
MePr	4.912 (6.501)	4.657 (6.525)	4.699 (6.616)	4.243 (6.696)
HiPr	0.398 (6.324)	0.100 (6.350)	1.061 (6.410)	1.492 (6.373)
Likelihood ins.	12.809** (6.277)	12.155* (7.150)	11.167 (9.458)	7.641 (9.620)
Optimism	-9.574* (5.589)	-12.281* (6.743)	11.532 (14.330)	-14.163 (30.600)
Pessimism	11.900 (7.288)	10.974 (7.355)	10.370 (14.029)	-13.416 (30.561)
Mixed	4.037 (6.764)	3.698 (6.789)	6.144 (7.065)	5.299 (7.047)
Convex	6.374 (17.822)	4.344 (18.033)	10.026 (18.842)	10.645 (18.684)
Concave	12.721 (8.673)	12.417 (8.700)	15.238* (8.848)	13.829 (8.815)
Constant	75.564*** (6.304)	77.548*** (6.579)	56.395*** (16.747)	83.116*** (31.805)
R <sup>2</sup>	0.101	0.108	0.094	0.100
Observations	172	172	172	172
Likelihood ins.	$\hat{s} < 1$	$\hat{s} < 1$	$\hat{\alpha} < 1$	$\hat{g} < 1$
Optimism	$\hat{c} > 0$ and $\hat{s} + \hat{c} \leq 1$	$\hat{c} > 0$ and $\hat{s} + \hat{c} \leq 1$	$\hat{\beta} < 1$	$\hat{\delta} > 1$
Parametric family	Neo-additive	Neo-additive	Prelec (1998)	Goldstein and Einhorn (1987)

Note: This table presents the estimates of the Ordinary Least Squares regression of the model  $Performance_i = \gamma_0 + \gamma_1 LowPr * Likelihoodins. + \gamma_2 LowPr * Optimism + \gamma_3 LowPr + \gamma_4 MePr + \gamma_5 HiPr + \gamma_6 Likelihoodins. + \gamma_7 Optimism + Controls' \Gamma + \epsilon_i$ , with  $E(\epsilon_i | MePr, LowPr, HiPr, Piecerate, Optimism, Likelihoodins., Controls, Mechanism) = 0$ . "Performance" is the number of correctly solved sums solved by a subject in the first part of the experiment, "Piecerate", "LowPr", "MePr" and "HiPr" are binary variables that indicate if a subject was assigned to the treatment offering a piece-rate or probability contracts implemented with low, medium or high probability, respectively. "Piecerate" is the benchmark of the regression. "Likelihood ins." is a binary variable that takes a value of one if the subject is likelihood insensitive and zero otherwise. "Optimism" is a binary variable that takes a value of one if the subject displays optimism and zero otherwise. Robust standard errors in parenthesis. \*\*\* denotes significance at the 0.01 level, \*\* denotes significance at the 0.05 level, \* denotes significance at the 0.1 level.

weighting functions proposed by [Prelec \(1998\)](#) and [Goldstein and Einhorn \(1987\)](#). As mentioned before, these functions contain, each, two parameters. One parameter mainly influences likelihood insensitivity and the other mainly influences optimism. On the basis of the magnitude of these parameters, I classify subjects according to their optimism as well as according to their sensitivity toward probabilities. As in the previous analysis, I include these alternative classifications in a regression relating performance, assignment to the treatments, optimism, and likelihood insensitivity. Columns (2) and (3) in [Table 6](#) present the results of the additional regressions. Altogether, the regression estimates corroborate the aforementioned results and, thus, the empirical validity of [Hypothesis 4](#) but not that of [Hypothesis 3](#). In particular, I find that likelihood insensitivity explains the documented performance differences between LowPr and Piecerate; subjects classified as likelihood insensitive and assigned to LowPr exhibit higher average performance levels as compared to subjects in Piecerate. Moreover, optimistic subjects assigned to LowPr do not exhibit performance differences with respect to subjects in the benchmark treatment.

## 6. Discussion and Conclusion

This paper introduced a novel contract designed to take advantage of the behavioral regularity that individuals distort probabilities. A theoretical framework and a laboratory experiment demonstrated that the proposed contract can generate higher output than standard piece-rate contracts. However, to achieve this result the principal is required specify a small probability that the performance-contingent outcome realizes. This implementation of the contract induces risk seeking attitudes in the agent and, as a consequence, generates a preference for stochastic compensation schemes. I show that the agent's insensitivity to likelihoods, the cognitive component of probability distortion, guarantees this result.

While the proposed contract can seem abstract, its incentives can be implemented using well-known tools of personnel economics. In the following, I discuss some ways to bring its incentives to practice.

- **Bonuses.** Consider a setting in which the agent's effort on a productive task and output relate stochastically. The principal can take advantage of this stochastic relationship by offering contracts that pay bonuses in the contingency that some output level is attained. Specifically, the principal can offer a contract paying a sizable bonus in the unlikely event that the highest levels of output realize. The agent with risk preferences characterized by RDU accepts these contracts as long as his perception about the likelihood of attaining the bonus is sufficiently overweighted. Furthermore, as the

findings of this paper show, this contract yields higher labor supply as compared to the hypothetical case in which the agent were incentivized with a cost-equivalent piece-rate.

- **Stock options.** A volatile firm can offer its workers a compensation plan that includes stock options. Naturally, the stock price at the time in which the option can be called is unknown when the contract is signed. This uncertainty can be used in the principal's advantage. First, as shown by Spalt (2013) the agent with RDU preferences will accept these contracts despite the firm being volatile, and thus being considerably risky. These risk seeking attitudes emerge because the agent overweights the probability associated to obtain large gains from calling the option. Second, as the results of this paper suggest, when the agent's effort shifts the *perceived* distribution of future stock prices, this contract generates higher labor supply than other standard performance-pay contracts. That is because the perceived contribution of the agent's effort to the probability of high future stock prices will be overweighted, inflating the perceived benefits of supplying high levels of effort under the proposed incentive scheme.
- **Auditing mechanisms.** Consider a setting in which the principal can choose among different auditing technologies. More advanced technologies, and also more expensive ones, allow the principal to more precisely monitor the agent's performance. These expensive technologies allow the principal to exactly link the agent's compensation to his performance. On the other hand, cheaper technologies are imperfect and, as a consequence, they entail that the exact labor supply of an agent is observed with some chance. The principal can take advantage of this source of uncertainty. The results of this paper show that when facing such trade-off, the principal can choose the cheaper technology and combine it with a large performance-contingent pay. When the agent with RDU preferences overweights the probability of being audited with the imperfect technology, he will be motivated to supply more output than if he were audited with the perfect technology.

A common property among these examples is that the incentives of the contract are brought to practice using natural sources of uncertainty: output realizations given effort, future stock prizes, and technological imperfections. Indexing the outcomes of the contract to uncertain and natural events allows the principal to circumvent the problem of lack credibility that might arise if she were to generate the contract's uncertainty using an artificial device, e.g. a roulette or dice. Assuring that the principal has no influence over the realization of uncertainty, and by extension over the outcome to be paid, allows her to more credibly commit to the contract and hence to successfully implement the desired incentives.

The present study has several limitations that open avenues for future research. First, it is assumed through the paper that the principal is fully informed about the agent's risk attitudes. Future research could focus on relaxing this assumption. Specifically, the false moral hazard model presented in Section 2 can be extended to incorporate adverse selection. In such a setting the uninformed principal can guarantee the effectiveness of the probability contract by screening RDU individuals, who exert more effort under the stochastic contract, from EUT individuals with concave utility, who will be harmed if incentivized with the proposed contract. To ensure truthful revelation of risk types, the principal can offer a menu of contracts and allow agents to self-select into the contract that best fits their risk attitudes. Within this menu at least two contracts should be included: a deterministic contract targeting agents with EUT preferences and concave utility, and a probability contract with a small probability targeting agents with RDU preferences and who overweight small probabilities. In this way the principal not only derives rents from offering the probability contract, as shown by this paper, but also avoids harming individuals with EUT preferences and risk averse attitudes.

Second, I studied the incentives of the contract in a setting of risk. A more complete understanding of these incentives should consider extending the present analysis to a setting of ambiguity. In such a setting the performance-contingent outcome of the contract is indexed to events realizing with unknown probability. Recent research suggests that individuals display more insensitivity toward ambiguity than toward risk (Baillon et al., 2018, Abdellaoui et al., 2011). Since likelihood insensitivity was found to be the main explanation for effectiveness of the contract, extending the analysis to a setting of ambiguity can potentially enhance the gains from its usage. Additionally, a setting of ambiguity would ease upon the practical implementation of the contract, as evidenced by the examples of practical implementations presented above.

Third, this paper considered a static setting. A more comprehensive investigation of the probability contract could examine its incentives in a setting of repeated interaction between principal and agent. On the theoretical side, it has been shown that, in settings with repeated interactions, optimal contracts exhibit properties that depend on the agent's expectation over the contracting span. When expectations are distorted, due to probability weighting, it is unclear whether these properties are enhanced or fade away and whether these conditions are more favorable for the implementation of stochastic contracts as compared to static settings. On the empirical side, an extension to a setup that admits repeated implementations of the contract would allow for a more robust analysis of its incentives. That is because such setting could shed light on whether and how probability weights are adjusted over time. In a relevant and related literature, it is suggested that individuals perceive probabilities differently when

they are described as compared to situations where they are experienced (Hertwig et al., 2004, Hau and Pleskac, 2008). If this is the case, the probability contract can have ambiguous effects in repeated settings: at first the contract can be effective, as shown by the present paper, but after some implementations the contract can become harmful to performance as long as the implemented probability is strongly underweighted once experienced.

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## Appendix A: Proofs

### Proposition 1

*Proof.* Assumption 2 is used to rewrite Equation (1) as follows:

$$U(t_d) = b(ay) - c\left(\frac{y}{\theta}\right). \quad (4)$$

The optimal output level  $y^*$  chosen by the agent when offered the contract  $t_d$  satisfies the following first order condition of Equation (4) with respect to  $y$ :

$$b'(ay^*)a - c'\left(\frac{y^*}{\theta}\right)\frac{1}{\theta} = 0. \quad (5)$$

Analogously, Assumption 2 and the equivalence  $A = \frac{a}{p}$  are used to rewrite Equation (3) as follows:

$$\mathbb{E}(U(t_s)) = pb\left(\frac{ay}{p}\right) - c\left(\frac{y}{\theta}\right). \quad (6)$$

The optimal output level  $y^{**}$  chosen by the agent when offered the contract  $t_s$  satisfies the following first order condition of Equation (6) with respect to  $y$ :

$$b'\left(\frac{ay^{**}}{p}\right)a - c'\left(\frac{y^{**}}{\theta}\right)\frac{1}{\theta} = 0. \quad (7)$$

Suppose that  $y^{**} < y^*$ , then Assumption 1 implies that for a given ability level  $\hat{\theta} \in [0, 1]$ :

$$y^{**} < y^* \Leftrightarrow c'\left(\frac{y^{**}}{\hat{\theta}}\right) < c'\left(\frac{y^*}{\hat{\theta}}\right). \quad (8)$$

Using Equation (5) and Equation (7) the equation above can be rewritten as:

$$b'\left(\frac{ay^{**}}{p}\right) < b'(ay^*). \quad (9)$$

Since  $\frac{ay}{p} \geq ay$  for any  $p \in (0, 1]$  and any  $y \in (0, \bar{y}]$ , then, by Assumption 3, it must be that  $b'\left(\frac{ay}{p}\right) \geq b'(ay)$  if  $b'' \geq 0$ , contradicting Equation (9). Hence, it must be that  $y^{**} \geq y^*$  if  $b'' \geq 0$ . Such contradiction is also achieved when unequal output levels are compared. To see how, consider arbitrary  $\epsilon > 0$ . Since  $\frac{a(y+\epsilon)}{p} > ay$  for any  $p \in (0, 1]$  and any  $y \in (0, \bar{y}]$ , then, by Assumption 3,  $b'\left(\frac{ay}{p}\right) \geq b'(ay)$  if  $b'' \geq 0$ , contradicting Equation (9). Hence, it must be that  $y^{**} \geq y^*$  if  $b'' \geq 0$ .

Following a similar procedure it is possible to demonstrate that  $y^{**} > y^*$  cannot hold if  $b'' < 0$  and that in such a case it must be that  $y^{**} \leq y^*$ . ■

### Proposition 2

*Proof.* Assumption 2 and the equivalence  $A = \frac{a}{p}$  are used to rewrite Equation (2) as follows:

$$RDU(t_s) = w(p)b\left(\frac{ay}{p}\right) - c\left(\frac{y}{\theta}\right). \quad (10)$$

The optimal output level  $y_R^{**}$  chosen by the agent with RDU preferences when working under  $t_s$  satisfies the following first order condition of Equation (10) with respect to  $y$ :

$$b'\left(\frac{ay_R^{**}}{p}\right)\frac{w(p)}{p}a - c'\left(\frac{y_R^{**}}{\theta}\right)\frac{1}{\theta} = 0. \quad (11)$$

Due to Assumption 4, there must exist an interval  $p \in (\hat{p}, 1)$  where probabilities are underweighted if  $\hat{p} < 1$ , as well as an interval  $p \in (0, \hat{p}]$  where probabilities are overweighted if  $\hat{p} > 0$ .

Consider first the interval whereby probabilities are underweighted  $p \in (\hat{p}, 1)$ . Suppose that for this interval  $y_R^{**} > y^*$ . Assumption 1 implies that for some given ability level  $\hat{\theta} \in [0, 1]$ , this relationship can be expressed as:

$$y_R^{**} \leq y^* \Leftrightarrow c'\left(\frac{y_R^{**}}{\hat{\theta}}\right) > c'\left(\frac{y^*}{\hat{\theta}}\right). \quad (12)$$

Using Equation (11) and Equation (5) I rewrite the above equation as

$$\frac{w(p)}{p}b'\left(\frac{ay_R^{**}}{p}\right) > b'(ay^*). \quad (13)$$

Since in the considered probability interval we have that  $\frac{w(p)}{p} < 1$ , then  $\frac{w(p)}{p}b'\left(\frac{ay}{p}\right) < b'(ay)$  for concave  $b$ , any output level  $y \in (0, \bar{y}]$ , and any piece-rate  $a > 0$ . Note that this conclusion also holds when unequal outcomes are considered. To see how, consider arbitrary  $\epsilon > 0$ , then it must be that  $\frac{w(p)}{p}b'\left(\frac{a(y+\epsilon)}{p}\right) < b'(ay)$  for concave  $b$ , any output level  $y \in (0, \bar{y}]$ , and any piece-rate  $a > 0$ . Hence, equation (13) cannot hold and we have arrived to a contradiction. Then, it must be that  $y_R^{**} < y^*$  if  $p \in (\hat{p}, 1)$  and  $b'' < 0$ . Since Proposition 1 demonstrates that  $y^{**} \leq y^*$  if  $b'' < 0$ , then the ranking  $y_R^{**} < y^{**} < y^*$  holds if  $t_s$  is offered with  $p \in (\hat{p}, 1)$  and the agent exhibits  $b'' < 0$ .

Next, consider the probability interval  $p \in (0, \hat{p}]$ . For this interval the ratio weights-to-probability displays  $\frac{w(p)}{p} \geq 1$ , which implies that both  $\frac{w(p)}{p}b'\left(\frac{ay}{p}\right) < b'(ay + \epsilon)$  and  $\frac{w(p)}{p}b'\left(\frac{ay}{p}\right) \geq$

$b'(ay + \epsilon)$  are possible for any  $y \in (0, \bar{y}]$ . Hence, and recognizing the result of Proposition 1, the relationships between output levels  $y_R^{**} \geq y^* > y^{**}$  and  $y^* > y_R^{**} \geq y^{**}$  are both possible in the considered probability interval whenever  $b'' < 0$ .

Suppose that  $y^* \geq y_R^{**}$ . Assumption 1 implies that for some given ability level  $\hat{\theta} \in [0, 1]$ , this relationship can be expressed as follows:

$$y^* \geq y_R^{**} \Leftrightarrow c' \left( \frac{y^*}{\hat{\theta}} \right) \geq c' \left( \frac{y_R^{**}}{\hat{\theta}} \right). \quad (14)$$

Equation (5) and Equation (11) are used to rewrite the above inequality as:

$$b'(ay^*) \geq \frac{w(p)}{p} b' \left( \frac{ay_R^{**}}{p} \right). \quad (15)$$

The validity of (15) is first analyzed at the two extremes of the considered probability interval. Since  $\frac{w(\hat{p})}{\hat{p}} = 1$ , then the inequality  $ay < \frac{a(y+\epsilon)}{\hat{p}}$  holds for any output level  $y \in (0, \bar{y}]$ , any piece-rate  $a > 0$ , and arbitrary  $\epsilon \geq 0$ . This conclusion together with the restriction  $b'' < 0$  yields that Equation (15) holds at the extreme  $p = \tilde{p}$ . Moreover, note that  $\lim_{p \rightarrow 0^+} \frac{w(p)}{p} b' \left( \frac{ay}{p} \right)$  yields an indeterminate form due to  $\lim_{p \rightarrow 0^+} b' \left( \frac{ay}{p} \right) = 0$ . Hence, to evaluate the left hand side of (15) as  $p \rightarrow 0^+$ , I use L'Hospital's rule as follows:

$$\lim_{p \rightarrow 0^+} \frac{w(p)}{p} b' \left( \frac{ay}{p} \right) = \lim_{p \rightarrow 0^+} \frac{\frac{d(b'(\frac{ay}{p}))}{dp}}{\frac{d(\frac{w(p)}{p})}{dp}} = \lim_{p \rightarrow 0^+} \left( \frac{w(p)}{p} \right)^2 \left( \frac{-b'' \left( \frac{ay}{p} \right) ay}{w(p) - w'(p)p} \right) = \infty. \quad (16)$$

The last equality is due to  $\lim_{p \rightarrow 0^+} \frac{w(p)}{p} = \infty$  and  $\lim_{p \rightarrow 0^+} w'(p)p = 0$ . Note that the expression,  $\lim_{p \rightarrow 0^+} w'(p)p$ , is also an indeterminate form and is thus also evaluated with L'Hospital's rule as follows:

$$\lim_{p \rightarrow 0^+} w'(p)p = \frac{\frac{dw'(p)}{dp}}{\frac{d(\frac{1}{p})}{dp}} = \lim_{p \rightarrow 0^+} -w''(p)p^2 = 0. \quad (17)$$

Where the last inequality in the above equation is requires  $w''(p) > -\infty$ . Therefore, the inequality in (15) does not hold as  $p \rightarrow 0^+$  and it must be that  $y^* < y_R^{**}$  as  $p \rightarrow 0^+$ .

Next, the behavior of the right hand side of Equation (15) as  $p$  increases within the interval  $p \in (0, \hat{p})$  is analyzed. To that end, compute the following derivative:

$$\frac{d \left( \frac{w(p)}{p} b' \left( \frac{ay}{p} \right) \right)}{dp} = \frac{(pw'(p) - w(p))}{p^2} b' \left( \frac{ay}{p} \right) - \frac{w(p)}{p^3 ay} b'' \left( \frac{ay}{p} \right). \quad (18)$$

Suppose that  $\frac{d \left( \frac{w(p)}{p} b' \left( \frac{ay}{p} \right) \right)}{dp} > 0$ , then, according to the above equation, positive changes in

probability within the considered interval must satisfy:

$$\frac{w'(p)}{\frac{w(p)}{p}} > 1 + \frac{b''\left(\frac{ay}{p}\right)\frac{ay}{p}}{b'\left(\frac{ay}{p}\right)}. \quad (19)$$

Assume for the moment that  $\hat{p} < \tilde{p}$ . Then for the considered interval,  $p \in (0, \hat{p})$ , the left hand side of (19) decreases as one considers positive changes in probability since  $\frac{d\left(\frac{w(p)}{p}\right)}{dp} > 0$  and  $w''(p) < 0$ . This implies that the largest value that  $\frac{pw'(p)}{w(p)}$ , the left hand side of Equation (19), attains is at  $p \rightarrow 0^+$ . To determine this value I use L'Hospital's rule:

$$\lim_{p \rightarrow 0^+} \frac{pw'(p)}{w(p)} = \lim_{p \rightarrow 0^+} \frac{w''(p)p}{w'(p)} + 1 = 1.$$

This result implies that the inequality presented in Equation (19) cannot hold for the considered probability interval, and instead it must be that  $\frac{d\left(\frac{w(p)}{p}b'\left(\frac{ay}{p}\right)\right)}{dp} < 0$  in  $p \in (0, \hat{p})$ .

So far, it has been established that  $\lim_{p \rightarrow 0^+} \frac{w(p)}{p}b'\left(\frac{ya}{p}\right) = \infty$  implying  $y^* < y_R^*$  as  $p \rightarrow 0^+$ . Also, that  $b'(ay^*) > b'\left(\frac{ay_R^*}{p}\right)$  at  $p = \hat{p}$ , implying  $y^* > y_R^*$  at that point. Finally, that  $\frac{d\left(\frac{w(p)}{p}b'\left(\frac{ay}{p}\right)\right)}{dp} < 0$  in  $p \in (0, \hat{p})$ . Altogether, these properties along with the continuity of  $w$  and  $b$  guarantee the existence of a  $p^* \in (0, \tilde{p})$  such that Equation (15) holds with equality. Hence  $t_s$  implemented with any  $p < p^*$  guarantees  $y^* < y_R^*$ .

To conclude the proof, note that to establish  $\frac{d\left(\frac{w(p)}{p}b'\left(\frac{ay}{p}\right)\right)}{dp} < 0$  it was assumed  $\hat{p} < \tilde{p}$ . Note that whenever  $\hat{p} \geq \tilde{p}$ , the existence of the probability  $p^*$  is also guaranteed, since any probability weighting function  $w(p)$  displaying  $\hat{p} > 0$  is always first concave at small probabilities and, if applies, becomes convex at higher probabilities. This property guarantees that  $\frac{d\left(\frac{w(p)}{p}b'\left(\frac{ay}{p}\right)\right)}{dp} < 0$  in  $p \in (0, \tilde{p})$  and thus that  $p^*$  making (15) hold with equality exists. However, for this case  $p^*$  is not unique since the left hand side of (19) is not always decreasing in the considered interval. Specifically, the left hand side of (19) is increasing at  $p \in (\tilde{p}, \hat{p})$ . Thus, for the case  $\hat{p} \geq \tilde{p}$  the probability  $p^*$  refers to the smallest possible value that makes Equation (15) bind with equality. ■

## Corollary 1 and Corollary 2

*Proof.* First it is shown that optimism is a necessary condition for  $\tilde{p} > 0$ . Let  $O(p)$  be a weighting function with the properties given in Assumption 4 and the additional assumption  $\tilde{p} = \{0, 1\}$ . This weighting function captures pessimism/optimism. Also, let  $L(p)$  be a weighting function with the properties given in Assumption 4 and the additional assumption  $\hat{p} = \tilde{p} = 0.5$ . This weighting function captures likelihood insensitivity. Suppose that

$w(p) = O(p)$ . Then, by construction, the existence of a non-empty interval  $p \in (0, \hat{p})$ , where probabilities are overweighted, is guaranteed only if  $O(p)$  displays  $\tilde{p} = 1$ , that is under optimism. In such case, the entire probability interval is overweighted.

Next, it is shown that likelihood insensitivity is a sufficient condition to generate a non-empty interval  $(0, \hat{p})$  where probabilities are overweighted. Let  $w(p) = O(p) + L(p)$ , with  $O(p)$  such that  $\tilde{p} = 0$ , that is the agent exhibits pessimism and likelihood insensitivity. Since  $\lim_{p \rightarrow 1^-} L'(p) = \infty$  and  $\lim_{p \rightarrow 1^-} O'(p) = \infty$ , then  $\lim_{p \rightarrow 1^-} w'(p) = \infty$ . Also, for the interval  $p \in (0.5, 1]$ , the probability weighting function exhibits  $w''(p) > 0$  since  $L''(p) + O''(p) > 0$ . At exactly  $p = 0.5$  the weighting function attains the value  $w'(0.5) = L'(0.5) + O'(0.5) = 0 + k$  for some  $k < \infty$ , given that  $O(p)$  exhibits  $\lim_{p \rightarrow -1} O'(1) = \infty$ ,  $\lim_{p \rightarrow 0^+} O'(p) \approx 0$ ,  $O''(p) < 0$ , and  $O'(p)$  is continuous, all of which guarantee  $|O'(0.5)| = k < \infty$ . Moreover,  $\lim_{p \rightarrow 0^+} w'(p) = \infty$  since  $\lim_{p \rightarrow 0^+} L'(p) = \infty$ . Hence, due to the continuity of the functions  $L'(p)$  and  $O'(p)$ , it must be that  $L''(p) + O''(p) < 0$  for a set of probabilities in the interval  $p \in (0, 0.5]$ . This implies the existence of an interior inflection point  $\tilde{p} \in (0, 1)$  generating  $w''(\tilde{p}) = 0$ , which at the same time entails the existence of an interval at which probabilities are overweighted  $p \in (0, \hat{p})$ .

Following a similar procedure it is straightforward to show the existence of  $\hat{p} \in (0, 1)$  when  $w(p) \equiv O(p) + L(p)$  and  $O(p)$  is a weighting function with the properties of Assumption 4 and  $\tilde{p} = 1$ , that is when the agent exhibits likelihood insensitivity and optimism. Therefore, regardless of  $\text{sgn}(O''(p))$ , the existence of a non-empty interval  $(0, \hat{p})$  where probabilities are overweighted is guaranteed for the weighting function  $w(p) = L(p) + O(p)$ . ■

## Appendix B: Agents with CPT preferences

In this Appendix, I compare the incentives generated by the two contracts when agents have risk preferences that can be characterized by CPT. I find that under mild additional conditions, the result stated in Proposition 2 holds and probability contracts can generate higher output than piece-rates. This finding is not surprising since CPT incorporates the key property of RDU that probability distortions are generated by probability transformations of cumulative probabilities. Therefore, when contracting with an agent with CPT preferences, the principal can specify a probability that induces risk seeking attitudes in the agent, generating a taste for stochastic contracts.

Agents with CPT preferences evaluate possible outcomes of the contract relative to a reference point  $r > 0$ . Outcomes below the reference point are coined *losses* and outcomes above it are *gains*. Typically,  $r$  represents a monetary amount that the worker expects to receive (Koszegi and Rabin, 2006) or a monetary amount that she owns (Kahneman et al., 1991). The novelty of CPT is that the agent can exhibit different risk preferences across these two domains. The evaluation of outcomes when  $t_s$  is offered and the equivalence  $A = \frac{a}{p}$  is assumed, is captured by a value function with the following properties:

**Assumption 5.**  $v(t_s)$  is the piecewise function,

$$v(t_s, r) = \begin{cases} b\left(\frac{ay}{p} - r\right), & \text{if } \frac{ay}{p} \geq r, \\ -\lambda b\left(r - \frac{ay}{p}\right), & \text{if } \frac{ay}{p} < r. \end{cases}$$

With  $r \geq 0$ ,  $\lambda > 1$ ,  $b'$  for all  $y \in [0, \bar{y}]$ ,  $b'' < 0$  if  $\frac{ay}{p} > r$ , and  $b'' > 0$  if  $\frac{ay}{p} < r$ .

The value function is an increasing function exhibiting concavity in the domain of gains and convexity in the domain of losses. This implies that the evaluation of outcomes relative to the reference point generates risk averse attitudes in the domain of gains and risk seeking attitudes in the domain of losses. Additionally, Assumption 5 states that the worker is loss-averse, which means that for him losses loom larger than gains. This property is represented by the parameter  $\lambda > 1$  which only enters the value function for the domain of losses.

As mentioned before, the CPT agent transforms the probabilities associated to the outcomes of the contract  $t_s$  using a probability weighting function. However, these transformations of probability can be different for gains and losses. Let  $w(p)$  be the probability weighting function used to transform probabilities in the domain of gains. This weighting function exhibits the properties from Assumption 4. Moreover, let  $z(p)$  be the probability weighting

function used to transform probabilities in the domain of losses. I assume that  $w(p)$  and  $z(p)$  relate through the duality  $z(p) = 1 - w(1 - p)$ . This duality has the implication that probability distortions that result from ordering the outcomes according to a rank from most-desirable to least-desirable is equivalent to the probability distortions that result from ranking outcomes from least-desirable to most-desirable.<sup>30</sup>

All in all, the utility of the agent with CPT preferences when offered  $t_s$  is equal to:<sup>31</sup>

$$CPT(t_s) = \begin{cases} w(p)b\left(\frac{ay}{p} - r\right) - c(e), & \text{if } \frac{ay}{p} \geq r \geq 0, \\ z(p)b\left(r - \frac{ay}{p}\right) - c(e), & \text{if } r > \frac{ay}{p} > 0. \end{cases} \quad (20)$$

The agent with CPT preferences supplies a level of output  $y_C^{**}$  satisfying the following system of equations:

$$\frac{a}{p}w(p)b'\left(\frac{ay_C^{**}}{p} - r\right) - c'\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta} = 0, \text{ if } \frac{ay}{p} \geq r, \quad (21)$$

$$\frac{a}{p}z(p)\lambda b'\left(r - \frac{ay_C^{**}}{p}\right) - c'\left(\frac{y_C^{**}}{\theta}\right)\frac{1}{\theta} = 0, \text{ if } \frac{ay}{p} < r. \quad (22)$$

Before proceeding to compare the optimal output level satisfying the above system of equations to that generated by  $t_d$ , it is worthwhile to describe the influence of loss aversion and reference points on  $y_C^{**}$ . Equation (22) shows that higher values of the loss aversion parameter,  $\lambda$ , yield higher output. This comparative static captures the notion that an agent is willing to supply more output to avoid experiencing losses. Additionally, the effect of a higher reference point  $r$  on output is ambiguous. On the one hand, higher reference points shift the schedules  $\frac{a}{p}w(p)b'\left(\frac{ay_C^{**}}{p} - r\right)$  and  $\frac{a}{p}z(p)\lambda b'\left(r - \frac{ay_C^{**}}{p}\right)$  to the right. Since these schedules are the positive components of Equation (21) and Equation (22), this comparative static is suggestive of higher reference points generating higher output. However, due to the fact that  $v$  can be convex, the solution to the system of equations composed by Equation (21) and Equation (22) features multiple equilibria, with equilibria at low levels of output becoming lower as  $r$  increases.<sup>32</sup> Thus, when the agent supplies output according to these

<sup>30</sup>Formally, an agent with CPT preferences facing a lottery  $(x_1, p_1; x_2, p_2; \dots; x_n, p_n)$  ranks the outcomes using an increasing arrangement  $x_1 < x_2 < \dots < x_{r-1} < r < x_{r+1} < \dots < x_n$  and evaluates the outcomes of the lottery relative to  $r$  through the function  $v(y, r)$ . The lottery outcomes  $x_{r+1}, \dots, x_n$  are gains and the outcomes  $x_1, \dots, x_{r-1}$  are losses. The individual assigns decision weights to gains in the following way  $\pi_n = w(p_n), \pi_{n-1} = w(p_{n-1} + p_n) - w(p_n), \dots, \pi_{r+1} = 1 - \sum_{j=r+1}^n w(p_j)$  and assigns decision weights to losses in the following way  $\pi_1 = z(p_1), \pi_2 = z(p_1 + p_2) - z(p_1), \dots, \pi_{r-1} = 1 - \sum_{j=r-1}^n z(p_j)$ .

<sup>31</sup>Note that there is an implicit assumption made throughout the three considered theories of risk: the monetary outcomes, whether evaluated according to final positions, as in EUT or RDU, or relative to a reference point, as in CPT, are represented by the same function  $b$ . This assumption is introduced to simplify the comparison between the studied contracts.

<sup>32</sup>To see how, consider first  $r = 0$ . For such case only the curve capturing the marginal utility of gains,

low-output equilibria, higher reference points yield lower output. The intuition of this result is that higher reference points can generate higher output up to a level after which they become unattainable and demotivate the agent.

Consider now the case in which the agent with CPT preferences works under  $t_d$ . Even though this piece-rate contract does not contain risk, which allows one to disregard probability weighting, it is assumed that the agent still makes decisions relative to a reference point (Kahneman et al., 1991). This assumption is consistent with the abundant evidence that in settings without risk, individuals exhibit reference-dependent preferences. For simplicity it is assumed that the agent has the same reference point for the two contracts. This assumption acquires strong validity in our setting inasmuch as both incentive contracts pay on expectation the same monetary amount and because the agent owns the same amount of money before being offered any of the contracts.

When offered  $t_d$ , the agent with risk less prospect theory preferences (Kahneman et al., 1991) supplies output according to  $y_C^*$  satisfying the following system of first-order conditions:

$$ab'(ay_C^* - rp) - c' \left( \frac{y_C^*}{\theta} \right) \frac{1}{\theta} = 0, \text{ if } ay \geq rp \quad (23)$$

$$\lambda ab'(rp - ay_C^*) - c' \left( \frac{y_C^*}{\theta} \right) \frac{1}{\theta} = 0, \text{ if } ay < rp. \quad (24)$$

Note that the reference point  $r$  becomes  $rp$  under the piece-rate due to the equivalence  $Ap = a$ , made in Section 2 and which we maintain here.

We are now in a position to compare the two contracts with respect to the output that they deliver. I first consider the case in which the agent is in the domain of gains,  $ay \geq rp$ . Corollary 3 demonstrates that probability contracts generate higher output than piece-rates.

**Corollary 3.** *Under Assumptions 1, 2, 4, and 5, for an agent with any ability level and a probability weighting function displaying  $\hat{p} > 0$  and  $w''(p) > -\infty$ , there exists a probability  $p^* \in (0, \hat{p})$  such that the contract  $t_s$  generates higher output if implemented with a probability satisfying  $p < p^*$  and the agent is in the domain of gains.*

*Proof.* Suppose that  $y_C^{**} < y_C^*$  for all  $p \in (0, 1]$ . Assumption 1 implies that for a given ability

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$\frac{a}{p}w(p)b'(\frac{ay}{p} - r)$ , is relevant to our case since only this curve attains non-negative values of output. The unique crossing point of  $\frac{a}{p}w(p)b'(\frac{ay}{p} - r)$  with  $c'(\frac{y}{\theta})$  determines the optimal output level. Now, consider a small increment of  $r = \epsilon$  for sufficiently small  $\epsilon > 0$ . Since higher values of  $r$  shift  $\frac{a}{p}w(p)b'(\frac{ay}{p} - r)$  to the right, the crossing point between this curve and  $c'(\frac{y}{\theta})$  also shifts to the right. However, now the curve  $\frac{a}{p}z(p)\lambda b'(r - \frac{ay}{p})$  can also attain positive values of output. Note that since  $b'' > 0$  if  $\frac{ay}{p} < r$  there are multiple crossings between  $\frac{a}{p}z(p)\lambda b'(r - \frac{ay}{p})$  and  $c'(\frac{y}{\theta})$ . One of these crossings is always smaller than the other, which, due to shift to the right of  $\frac{a}{p}z(p)\lambda b'(r - \frac{ay}{p})$  as  $r$  increases, becomes lower with higher reference points.

level  $\hat{\theta} \in [0, 1]$ :

$$y_C^{**} < y_C^* \Leftrightarrow c' \left( \frac{y_C^{**}}{\hat{\theta}} \right) < c' \left( \frac{y_C^*}{\hat{\theta}} \right). \quad (25)$$

Using Equation (21) and Equation(23), the above inequality can be rewritten as:

$$\frac{a}{p} w(p) b' \left( \frac{a y_C^{**}}{p} - r \right) < a b' (a y_C^* - r p). \quad (26)$$

In the domain of gains, that is as long as  $\frac{ay}{p} \geq r$ , the inequality  $\frac{ay}{p} - r > ay - rp$  holds for any  $y \in (0, \bar{y}]$ ,  $p \in (0, 1)$ , and  $a > 0$ . Assumption 5 states that  $b'' < 0$  if  $\frac{ay}{p} > r$ , then it must be that  $b' \left( \frac{ay}{p} - r \right) < b'(ay - rp)$ . This inequality also holds for unequal output values  $y$ . To see how note that  $b' \left( \frac{ay}{p} - r \right) < b'(a(y + \epsilon) - rp)$  holds for any  $y \in (0, \bar{y}]$ ,  $p \in (0, 1)$ , any  $a > 0$  and arbitrary  $\epsilon > 0$ . Therefore, the inequality in (26) holds for any probability in  $p \in [\hat{p}, 1]$  since for this interval  $\frac{w(p)}{p} < 1$  and, as already established,  $b' \left( \frac{ay}{p} - r \right) < b'(a(y + \epsilon) - rp)$  for arbitrary  $\epsilon > 0$ . Thus, (26) is corroborated in  $p \in [\hat{p}, 1]$  by letting  $y_C^{**} = y$  and  $y_C^* = y_C^{**} + \epsilon$ .

Consider now the probability interval  $p \in (0, \hat{p}]$ . While  $b' \left( \frac{ay}{p} - r \right) \leq b'(a(y + \epsilon) - rp)$  remains valid, now the ratio of probability weights-to-probability exhibits  $\frac{w(p)}{p} > 1$ . Therefore, when the ratio  $\frac{w(p)}{p}$  is sufficiently large, the inequality in (26) can be invalidated. First, it is established whether at the extremes of the interval  $p \in (0, \hat{p}]$  Equation (26) can be invalidated. As  $p \rightarrow \hat{p}$ , then  $\frac{w(\hat{p})}{\hat{p}} = 1$  and the inequality in (26) holds. Instead, as  $p \rightarrow 0^+$  that  $\lim_{p \rightarrow 0} a b'(a y_C^* - r p) = y^* \leq \bar{y}$ , meaning that the right hand side of Equation (26) approaches the left hand side of Equation (5). Thus, as  $p \rightarrow 0^+$  the inequality becomes similar to that of Proposition 2. From the proof of Proposition 2 is established that  $\lim_{p \rightarrow 0^+} \frac{a}{p} w(p) b' \left( \frac{ay}{p} \right) = \infty$ , a conclusion that can be extended without loss of generality to  $\lim_{p \rightarrow 0^+} \frac{a}{p} w(p) b' \left( \frac{ay}{p} - r \right) = \infty$  for  $r \geq 0$ . Hence, the inequality in (26) does not hold as  $p \rightarrow 0^+$ .

Furthermore, the behavior of  $\frac{a}{p} w(p) b' \left( \frac{ay}{p} - r \right)$  as  $p$  changes in  $p \in (0, \hat{p})$  is the same as that of  $\frac{a}{p} w(p) b' \left( \frac{ay}{p} \right)$ , which in Proposition 2 was established to exhibit  $\frac{d \frac{a}{p} w(p) b' \left( \frac{ay}{p} \right)}{dp} < 0$ . This implies the existence of a  $p^* \in (0, \hat{p}]$  such that Equation (26) holds with equality. Thus,  $y_C^{**} > y_C^*$  is implemented as long as  $t_s$  with a probability  $p < p^*$  ■

As with RDU preferences, the principal generates more output when she offers the probability contract with a probability that is sufficiently overweighted by the agent. The overweighting of small probabilities induces risk-seeking attitudes in the agent, which, when sufficiently strong, can outweigh the risk averse attitudes generated by concavity of the value function in the domain of gains. These global risk-seeking attitudes motivate the agent with CPT preferences to deliver more output under the probability contract.

Consider now the case in which the agent is in the domain of losses  $\frac{ay}{p} < r$ . The result that the probability contract generates higher output than piece-rates also holds for this domain, but under less stringent conditions. Corollary 4 demonstrates this result.

**Corollary 4.** *Under Assumptions 1, 2, 4, and 5, for an agent with any ability level and who transforms probabilities with a weighting function  $z(p) \equiv 1 - w(1 - p)$  with the properties  $z''(p) > -\infty$  and  $\hat{p} > 0$ , the contract  $t_s$  generates higher output if the specified probability satisfies  $p < \hat{p}$  and the agent is in the domain of losses.*

*Proof.* Suppose that  $y_C^{**} \leq y_C^*$ . Assumption 1 implies that for a given ability level  $\hat{\theta} \in [0, 1]$ :

$$y_C^{**} \leq y_C^* \Leftrightarrow c' \left( \frac{y_C^{**}}{\hat{\theta}} \right) < c' \left( \frac{y_C^*}{\hat{\theta}} \right). \quad (27)$$

Using Equation (22), Equation (24), and the duality  $z(p) = 1 - w(1 - p)$  the above inequality can be rewritten as:

$$\frac{(1 - w(1 - p))}{p} b' \left( r - \frac{a}{p} y_C^{**} \right) \leq b'(rp - ay_C^*). \quad (28)$$

For the loss domain,  $r > \frac{ay}{p}$ , the inequality  $r - \frac{ay}{p} > rp - ay$  holds for any  $y \in (0, \bar{y}]$ , any  $r > 0$ , any  $a > 0$ , and any  $p \in (0, 1)$ . Assumption 5 states that  $b'' > 0$  if  $r > \frac{ay}{p}$ , then it must be that  $b'(r - \frac{a}{p}y) \geq b'(rp - ay)$ . Such inequality continues to hold when  $b'(r - \frac{a}{p}(y + \epsilon)) > b'(rp - ay)$  for arbitrary  $\epsilon > 0$ . Moreover,  $\frac{1-w(1-p)}{p} \geq 1$  for  $p \in (0, \tilde{p}]$ . Altogether that  $b'(r - \frac{a}{p}(y + \epsilon)) > b'(rp - ay)$  for arbitrary  $\epsilon > 0$  and  $\frac{1-w(1-p)}{p} \geq 1$ , imply that Equation (28) does not hold when  $y_C^* := y + \epsilon$  and  $y_C^{**} := y_C^*$ . Hence, it must be that  $y_C^{**} > y_C^*$  if  $p \in (0, \hat{p})$  and  $\frac{ay}{p} < r$  are assumed. ■

An agent with CPT preferences exhibits a convex value function in the domain of losses. This curvature generates risk-seeking attitudes, which favors labor supply under the probability contract. To maintain these favorable risk attitudes, the principal should avoid choosing probabilities that induce risk-averse attitudes. This could be done implementing any  $p \in (0, \tilde{p}]$ . Since  $p^* < \hat{p}$  the result in Corollary 4 is guaranteed when  $p \in (0, p^*)$ .

## Appendix C: The principal's problem

The purpose of this Appendix is to complement the theoretical model presented in Section 2. To that end, I present a solution to the principal's problem which encompasses the incentive compatibility and participation constraints of the agent. While the first constraint was the main focus of Section 2, the participation constraint and the objective function of the principal was not taken into account. The results from the analysis presented in this Appendix exhibits similar properties as the solution found in Proposition 2. Specifically, it is found that it is optimal for the principal to offer the probability contract with probabilities that are strongly overweighted by the agent, which generate a taste for stochastic contracts, and, thus, incentivize higher labor supply as compared to a piece-rate.

Assume that the agent's risk preferences are characterized by RDU. That is, he distorts cumulative probabilities using the weighting function  $w(p)$  described by Assumption 4. Also, to simplify the analysis and to be able to find a tractable solution, it is assumed that the agent has a power utility function as follows:

**Assumption 6.** *Let  $b(t) = t^\rho$  where  $\rho \in (0, 1]$ .*

I study a setting in which the risk-neutral principal offers  $t_s$  and determines the probability  $p \in (0, 1]$ . As in the main body of the paper, the equivalence  $A = \frac{a}{p}$  is assumed. Thus, the principal can choose among contracts that are cost-equivalent but that differ on the amount of risk that will be faced by the agent. When introducing risk is disadvantageous, she can choose  $p = 1$  and the agent is compensated according to a piece-rate.

All in all, the principal's objective is to minimize the amount paid when using  $t_s$ , subject to the participation constraint and the incentive compatibility of the agent. This problem is formally described next:

$$\begin{aligned}
 \min_p \quad & Apy \\
 \text{s.t.} \quad & w(p)b(Ay)^\rho - c\left(\frac{y}{\theta}\right) \geq 0, \\
 & \max_y w(p)b(Ay)^\rho - c\left(\frac{y}{\theta}\right)
 \end{aligned} \tag{29}$$

The solution to the principal's problem is presented in Proposition 3. The solution features multiple choices for the principal, which depend on the desired probability as well as on the location of  $\hat{p}$  with respect to  $\tilde{p}$  in the agent's weighting function.

**Proposition 3.** Under Assumptions 1, 2, 4, and 6, the solution to the principal's program is

$$p^{opt} = \begin{cases} p^{**} & \text{if } p < \tilde{p}, \\ \hat{p} & \text{if } \tilde{p} < \hat{p} \text{ and } p > \tilde{p}, \\ 1 & \text{if } \tilde{p} > \hat{p} \text{ and } p > \tilde{p}. \end{cases}$$

Where  $p^{**}$  satisfies  $p^{**} = \frac{\rho(p^{**})}{w'(p^{**})}$  and the probability weighting function  $w(p)$  exhibits the properties  $\hat{p} \in (0, 1)$  and  $w''(p) > -\infty$ .

*Proof.* Recognizing the equivalence  $A = \frac{a}{p}$  the Lagrangian of the principal program is:

$$\mathcal{L} = ay - \nu_1 \left( w(p) \left( \frac{ay}{p} \right)^\rho - c \left( \frac{y}{\theta} \right) \right) - \nu_2 \left( \frac{arw(p)}{p} \left( \frac{ay}{p} \right)^{\rho-1} - c' \left( \frac{y}{\theta} \right) \frac{1}{\theta} \right). \quad (30)$$

Where  $\nu_1$  and  $\nu_2$  are the multipliers of the participation constraint and incentive compatibility constraint, respectively. The first-order condition of the Lagrangian in (30) with respect to  $p$  is:

$$\begin{aligned} -\nu_1 \left( w'(p) \left( \frac{ay}{p} \right)^\rho - \frac{a\rho y w(p)}{p^2} \left( \frac{ay}{p} \right)^{\rho-1} \right) \\ - \nu_2 \left( \frac{a\rho w'(p)}{p} \left( \frac{ay}{p} \right)^{\rho-1} - \frac{a\rho w(p)}{p^2} \left( \frac{ay}{p} \right)^{\rho-1} - \frac{a^2 \rho (\rho-1) y w(p)}{p^3} \left( \frac{ay}{p} \right)^{\rho-2} \right) = 0. \end{aligned} \quad (31)$$

After some manipulations, Equation (31) can be rewritten as:

$$- \left( \frac{ay}{p} \right)^\rho \left( \nu_1 + \frac{\nu_2 \rho}{y} \right) \left( w'(p) - \frac{\rho w(p)}{p} \right) = 0 \quad (32)$$

From Equation (32) it can be established that when  $\nu_1 > 0$  or if  $\nu_2 > 0$  the solution of the Lagrangian is given by the probability satisfying  $p^{**} = \frac{\rho w(p^{**})}{w'(p^{**})}$ .

Define  $g(p) := \frac{\rho w(p)}{w'(p)}$ . Note that  $g(0) = 0$  since  $\lim_{p \rightarrow 0^+} w'(p) = \infty$  and  $w(0) = 0$ . Also,  $g(1) = 0$  since  $\lim_{p \rightarrow 1^-} w'(p) = \infty$  and  $w(1) = 1$ . Similarly,  $g(\tilde{p}) = \infty$  since  $\lim_{p \rightarrow \tilde{p}} w'(p) = 0$ . Moreover, note that  $g(p)$  is increasing in  $p \in [0, \tilde{p})$ , due to the fact that  $w''(p) > 0$  and  $g(p)$  is decreasing in  $p \in [1, \tilde{p}]$ , due to the fact that  $w''(p) < 0$  in  $p \in [1, \tilde{p}]$ .<sup>33</sup> These properties of  $g(p)$  guarantee the existence of the fixed-point  $p^{**} = g(p^{**})$  in  $p \in (0, 1)$ . To see how, note that  $p$  is

<sup>33</sup>Since  $g'(p) = \rho - \frac{\rho w(p) w''(p)}{(w'(p))^2}$ , a sufficient condition for  $g'(p) > 0$  is that  $w''(p) < 0$ , and a necessary condition for  $g'(p) < 0$  is that  $w''(p) > 0$ .

an increasing function that attains a minimum at  $p = 0$  and a maximum at  $p = 1$ . Moreover, it was already established that  $g(p)$  attains values  $g(0) = 0$ ,  $g(1) = 0$ , and  $g(\tilde{p}) = \infty$ , and it was also established that  $g(p)$  increases in  $p \in (0, \tilde{p})$  and decreases in  $p \in (\tilde{p}, 1)$ . Then, due to the continuity of  $w(p)$ , there must exist a probability  $p \in (0, 1)$  where  $p$  and  $g(p)$  intersect.

Next, it is shown that  $p^{**}$ , the probability at which the functions  $p$  and  $g(p)$  intersect, is located in the interval  $p \in (0, \tilde{p})$ . Note that  $g(0) = 0$ , which entails that  $g(p)$  and  $p$  have the same departing point. Since  $\lim_{p \rightarrow 0^+} g'(p) = \rho - \frac{\rho w(p)w''(p)}{(w'(p))^2} = \rho$ , due to the property  $|w''| < \infty$  and  $\lim_{p \rightarrow 0^+} w'(p) = \infty$ , the slope of  $g(p)$  is lower than that of  $p$  as  $p \rightarrow 0^+$ . That  $g(p) < p$  for small  $p$  and  $g(\tilde{p}) = \infty$ , it must be that the functions  $g(p)$  and  $p$  intersect at some probability  $p \in (0, \tilde{p}]$ .

Finally it is investigated whether  $p^{**}$  is a solution to the principal's program for all values of  $p$ . To do so, I investigate the shape of the Lagrangian in Equation (30). The second-order condition of this Lagrangian is:

$$\frac{a\rho y}{p^2} \left(\frac{ay}{p}\right)^{\rho-1} \left(\nu_1 + \frac{\nu_2\rho}{y}\right) \left(w'(p) - \frac{\rho w(p)}{p}\right) - \left(\frac{ay}{p}\right)^\rho \left(\nu_1 + \frac{\nu_2\rho}{y}\right) \left(w''(p) - \frac{p\rho w'(p) - rw(p)}{p^2}\right) \quad (33)$$

Equation (33) becomes positive when evaluated at  $p^{**}$  if  $w''(p) < 0$ . Hence  $p^{**}$  is a solution of the program as long as  $p \in (0, \tilde{p})$ . In contrast, if  $w''(p) > 0$ , the second order condition in (33) becomes negative, implying that the objective function attains a minimum value at one of the extremes,  $p = \tilde{p}$  or  $p = 1$ . Hence, unless  $p^{**} = 1$  or  $p^{**} = \hat{p}$ , there exists multiple solutions to the principal's problem.

Note that according to Assumption 4,  $w(p)$  might display  $\hat{p} \neq \tilde{p}$ . Let  $\hat{p} > \tilde{p}$ . In such case, for  $p \in (0, \tilde{p})$ , the solution  $p = p^{**}$  can be implemented. Instead, for  $p \in [\tilde{p}, 1]$  the solution is either at  $p = \tilde{p}$  or at  $p = 1$ . Since at  $p = \tilde{p}$  probabilities are overweighted, the incentive compatibility and participation constraints of the program presented become larger than at  $p = 1$ , which implies that at  $p = \tilde{p}$  the Lagrangian attains a lower value. Thus, the principal chooses  $p = \tilde{p}$  if  $p \in (\tilde{p}, 1)$  and  $\hat{p} > \tilde{p}$ .

Let now  $\hat{p} \leq \tilde{p}$ . As before, for the interval  $p \in (0, \tilde{p})$ ,  $p = p^{**}$  is implemented even though this can imply that probabilities are going to be implemented such that  $\frac{w(p)}{p} < 1$ . Moreover, for the interval  $p \in [\tilde{p}, 1]$ , the solution to (19) is  $p = 1$  since  $p = \tilde{p}$  yields  $\frac{w(p)}{p} < 1$  which leads to lower values of the incentive compatibility and participations constraints than those implied by  $p = 1$ . ■

Proposition 3 shows that the principal's program has multiple solutions. This is because

$w(p)$ , which codetermines the shape of the agent's objective function, can be concave, convex, or both. When concavity and convexity coexist, an interior solution as well as a knife-edge solution to the principal's program emerge. Specifically, for  $p \in (0, \tilde{p}]$  the principal sets  $p = p^{**}$ . When the weighting function of the agent displays  $\tilde{p} < \hat{p}$ , this solution always induces an overweighting of probabilities in the agent, which generates a global preference for risk, enhancing the agent's performance when he works under the probability contract. For the case  $\tilde{p} \geq \hat{p}$  a similar conclusion is reached; whenever  $p^{**} \in (0, \hat{p})$  the principal implements  $t_s$  with a probability that is overweighted by the agent. However, for this case implementing the contract at  $p^{**}$  can lead to underweighting of probabilities as long as  $p^{**} \in [\hat{p}, \tilde{p}]$ , which induces global risk aversion in the agent. Such implementation of the contract yields lower labor supply when  $t_s$  is offered as compared to offering a piece-rate contract  $t_d$ . The intuition behind this counterintuitive result is that when the principal is restricted to implement the contract in the interval  $p < \tilde{p}$ , she can incentivize labor supply choosing  $p^{**} \in [\hat{p}, \tilde{p}]$  while offering the contract with a lower probability  $p < \hat{p}$  is not a possibility since it is not incentive feasible, i.e the contract will not be accepted by the agent or it does not incentivize the agent to leverage optimal output.

Additionally, for  $p \in (\tilde{p}, 1]$  the optimal choice for the principal lies in the set  $p = \{\tilde{p}, 1\}$ . When the weighting function exhibits  $\tilde{p} < \hat{p}$ , setting  $p = \tilde{p}$  is preferred since this could induce overweighting of this probability in the agent, which incentivizes high labor supply. Instead, when  $\hat{p} \geq \tilde{p}$ , setting  $p = 1$  is preferred, since implementing any  $p \in (\tilde{p}, 1]$  would induce underweighting of probabilities in the agent, demotivating labor supply.

I conclude this Appendix by summarizing its main results. I find that the principal's optimal choice consists on multiple solutions that either induce overweighting of probabilities in the agent or imply implementing the piece-rate contract.<sup>34</sup> This solution corroborates the result in Proposition 2: whenever possible the principal chooses a probability that generates a taste for risky incentive contracts in the agent, such a preference for risky contracts enhance labor supply.

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<sup>34</sup>Note that for the special case in which  $\tilde{p} > \hat{p}$  and  $p^{**} \in [\hat{p}, \tilde{p}]$ , the principal can also choose  $p = 1$  when she is not restricted to choose in the interval  $p \in (0, \tilde{p})$ . A simple profit maximization argument entails that  $p = 1$  is preferred to  $p = p^{**}$ .

## Appendix D: Instructions

This is an experiment in the economics of decision-making. The instructions are simple and if you follow them carefully and make certain decisions, you might earn a considerable amount of money, which will be paid to you via bank transfer at the end of the experiment. The amount of money that you earn will depend on your decisions and effort, and partly on chance. Once the experiment has started, no one is allowed to talk to anybody other than the experimenter. Anyone who violates this rule will lose his or her right to participate in this experiment. If you have further questions when reading these instructions please do not hesitate to raise your hand and formulate the question to the experimenter.

The experiment consists of two parts. Your earnings in part one or part two of the experiment will be chosen at the end of the experiment and become your final earnings. Whether the earnings of part one or the earnings of part two will be your final earnings will be established by roll of a die.

### Part one

In this part of the experiment your task is to complete summations. Your earnings in this part of the experiment depend only on the number of correct summations that you deliver. You need to complete as many summations as you can in 10 rounds, each round lasts four minutes. In other words you will have a total of 40 minutes to complete as many summations as you can.

Each summation consists of five-two digit numbers. For example  $11+22+33+44+55=?$  Once you know the answer to the sum of these five two digit numbers, input the answer in the interface, Click OK, and a new set of numbers will appear on your screen.

For your better understanding you will face with two examples next.

[Examples displayed]

The previous examples show what you have to do in this part of the experiment. The only thing left to be explained is to specify how you are going to earn money by completing the summations.

**Piecerate Treatment Payment rule:** In this part of the experiment each correct

summation will add 25 Euro cents to your experimental earnings.

Remember: you have 40 minutes to complete summations, and only correct summations will count towards your earnings at a rate of 25 euro cents each. If you understood these instructions Press "OK". When everyone is ready we will start with this part of the experiment.

**LowPr Treatment Payment rule:** In this part of the experiment 1 out of all the 10 rounds will be randomly chosen. The specific round is chosen at random by the computer at the end of this part of the experiment. This is, once you completed summations in all the 10 rounds, only the correct summations in a randomly chosen round will count towards your earnings at a rate of 250 euro cents per correct summation.

Remember: you have 40 minutes to complete summations, and only correct summations in 1 specific round, chosen randomly by the computer at the end of the experiment, will count towards your earnings at a rate of 250 euro cents each. If you understood these instructions Press "OK". When everyone is ready we will start with this part of the experiment.

**MePr Treatment Payment rule:** In this part of the experiment 3 out of all the 10 rounds will be randomly chosen. The specific rounds are chosen at random by the computer at the end of this part of the experiment. This is, once you completed summations in all the 10 rounds, only the correct summations in that randomly chosen round will count towards your earnings at a rate of 85 euro cents per correct summation.

Remember: you have 40 minutes to complete summations, and only correct summations in 3 specific rounds, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 85 euro cents each. If you understood these instructions Press "OK". When everyone is ready we will start with this part of the experiment.

**HiPr Treatment Payment rule:** In this part of the experiment 5 out of all the 10 rounds will be randomly chosen. The specific rounds are chosen at random by the computer at the end of this part of the experiment. This is once you completed summations in all the 10 rounds, only the correct summations in that randomly chosen round will count towards your earnings at a rate 50 euro cents per correct summation.

Remember: you have 40 minutes to complete summations, and only correct summations in 5 specific rounds, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 50 euro cents each. If you understood these instructions Press "OK". When everyone is ready we will start with this part of the experiment.

## Part two

In this part of the experiment your task is to choose among two possible alternatives. Your earnings on this part of the experiment depend on how good your choices are. Particularly, you will face with 11 decision sets. In each of these sets you need to choose between the option L, that delivers a fixed amount of money, and the option R that is a lottery between two monetary amounts. Each decision set contains six choices.

Be Careful! Every time you make a choice between L and R, the monetary prizes of the options are going to change and you ought to make a choice again. One of your choices will be randomly picked by the computer, will be played and its realization will count towards your earnings for this part of the experiment. You will be faced with one example next. [Example displayed]

If it is clear what you have to do in this part of the experiment. Press "OK" to start, once everyone is ready this part of the experiment will begin.

## Survey

- Gender:
- Age:
- What is your education level? (Bachelor, Exchange, Pre-Master, Master, PhD):
- What is the name of your program of studies?
- How difficult did you find the task? (where 1 stands for easy and 5 for Very difficult)
- Rate how confident you are that you can do the task good enough so you can be in the top half of performers in this group as of now. (1-Not confident, 10- Very confident)
- Are you any good at adding numbers? (1-Not good at all, 10-Very good)
- Are you generally a person who is fully prepared to take risks or do you try to avoid taking risks?
- Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks".

- People can behave differently in different situations. How would you rate your willingness to take risks while driving? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks in financial matters? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks with your health? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks in your occupation? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"
- How would you rate your willingness to take risks in your faith in other people? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks"

## Appendix E: Performance Beliefs

Section 5.1 presents empirical evidence supporting Hypothesis 1. Specifically, the data suggest that subjects assigned to LowPr display higher average performance than subjects assigned to Piecerate, MePr, or HiPr. The aim of this Appendix is to investigate whether subjects anticipate the effect of the incentives included in probability contracts on their own performance. To that end, I analyze the subjects' beliefs about their own performance in the real-effort task across treatments. The idea is that if subjects internalize the incentives included in those treatments, their beliefs should reflect the performance differences documented in Section 5.1.

Table 7 presents the descriptive statistics of performance beliefs by treatment. The data suggest that subjects in LowPr, MePr and HiPr display average beliefs that are statistically indistinguishable.<sup>35</sup> Likewise, I find that subjects assigned to Piecerate display similar average beliefs as those of subjects assigned to LowPr, MePr, and HiPr.<sup>36</sup> These results suggest that subjects did not anticipate the effect that the different probabilities had on their own performance.

Table 7: Descriptive statistics of performance beliefs by treatments

Treatment	LowPr	MePr	HiPr	Piecerate	Total
Mean	83.86	82.025	74.022	73.177	78.123
Median	75	80	75	64	74.500
St. Dev.	40.864	40.156	36.139	43.318	40.147
N	43	40	44	45	172

size Note: This table presents the average, median and standard deviations of performance beliefs by treatment. A performance belief is the estimate of a subject about the number of correct summations solved in the real-effort task.

I also perform a regression analysis to account for factors other than the treatment assignment that could drive these results. In particular, I include control variables in the regression that capture the shape of the utility function and the shape of the probability weighting function of subjects. Thus, this analysis can show whether the above conclusion of no difference between the treatments is driven by a potential confound between risk variables

<sup>35</sup>The t statistics of these comparisons are: MePr vs. LowPr ( $t(80.749) = 0.206, p = 0.837$ ), HiPr vs. MePr ( $t(78.819) = 0.956, p = 0.3418$ ), and LowPr vs. HiPr treatments ( $t(83.241) = 1.1885, p = .1190$ )

<sup>36</sup>The statistics of these t-tests are ( $t(85.98) = 1.190, p = 0.1186$ ), ( $t(82.843) = 0.976, p = 0.331$ ), and ( $t(84.91) = -0.10, p = 0.920$ ), respectively.

and treatment assignment, generated by an incorrect randomization.

Table 8 presents the OLS estimates of different statistical models. All regression estimates corroborate the aforementioned results. Specifically, the coefficients associated to the MePr, HiPr and LowPr treatments are not significant, suggesting no statistical differences between the average beliefs of subjects assigned to those treatments and those of subjects assigned to Piecerate. Furthermore, there is no evidence to reject the null hypothesis that the coefficients of LowPr and MePr are equal ( $F(1, 160) = 0.29$ ), as well as no evidence to reject the null hypothesis that the coefficients of LowPr and HiPr are equal ( $F(1, 160) = 0.09$ ).

All in all, the beliefs data suggest that subjects do not internalize the incentives included in the probability contract. This finding implies that subjects assigned to LowPr exhibit a steep gap between their beliefs and their performance on the task. I conjecture that such gap can be explained in light of the findings of Berns et al. (2008), who show that probability distortions are a perceptual phenomenon and that cognitive processes of higher consciousness, that allow individuals to internalize these distortions, are not involved in their formation. This neurobiological foundation of probability distortion can explain that subjects are unlikely to understand how an assignment to a treatment representing the probability contract can influence their performance.

Understanding the reasons behind the gap between performance and performance beliefs is beyond the scope of this paper and requires methodologies that allow the researcher to study in more detail the cognitive processes underlying probability judgments.

Finally, note that the observed gap between the subjects' performance and their beliefs can explain the scarce occurrence of stochastic contracts. If a principal does not anticipate the non-monetary incentives of the contract, she may be inclined to choose simpler payment modalities. Furthermore, even when the principal is informed about the way in which agents distort probabilities, she might correctly believe, just as the results of this Appendix show, that the agent is not going to internalize the non-monetary incentives included in the probability contract. However, this belief can be erroneously interpreted, leading her to conclude that such lack of anticipation entails similar performance across the contracts. The importance of this paper is showing not only that the proposed contract is more effective, but also that its incentives have a perceptual foundation and are unanticipated by the worker.

Table 8: Regression of performance beliefs on treatments

	(1)	(2)	(3)	(4)
	Beliefs	Beliefs	Beliefs	Beliefs
LowPr	10.683 (8.977)	10.675 (9.366)	10.205 (9.434)	9.098 (9.336)
MePr	8.847 (9.055)	8.262 (9.285)	7.929 (9.327)	7.651 (9.214)
HiPr	0.845 (8.452)	-0.140 (8.294)	0.348 (8.427)	-3.373 (8.452)
Concave		26.405* (14.354)	24.950* (14.470)	24.873* (13.872)
Convex		10.174 (17.330)	9.955 (18.148)	4.495 (16.378)
Linear		5.057 (8.177)	3.726 (8.251)	
LS			3.408 (6.883)	
US			-3.818 (6.159)	
Possibility				16.649 (11.371)
Certainty				17.551 (11.135)
Constant	73.178*** (6.461)	67.485*** (8.764)	69.583*** (9.755)	57.489*** (11.152)
R <sup>2</sup>	0.014	0.037	0.041	0.050
Observations	172	172	172	172

Note: This table presents the estimates of the Ordinary Least Squares regression of the model  $Belief_i = \theta_0 + \theta_1 LowPr + \theta_2 MePr + \theta_3 HiPr + Controls' \Lambda + \epsilon_i$ , with  $E(\epsilon | MePr, LowPr, HiPr, Controls) = 0$ . “Beliefs” is the predicted number of correctly solved sums by a subject in the first part of the experiment, “LowPr”, “MePr” and “HiPr” are dummy variables that capture whether the subject was assigned to the treatment offering the probability contract with low, medium or high probability, respectively. The controls considered in this model are “LS” a binary variable that takes a value of one if a subject has a weighting function with lower subadditivity and zero otherwise, “US” a binary variable that takes a value of one if a subject has a weighting function with upper subjectivity and zero otherwise, “Possibility” a binary variable takes a value of one if a subject has a weighting function with the possibility effect being stronger than the certainty effect and zero otherwise. “EQ” a binary variable taking a value of one if a subject has a weighting function with the having the same magnitude as the possibility effect and zero otherwise. “Concave” a binary variable that takes a value of one if a subject has a concave utility function and zero otherwise, “Convex” a binary variable that takes a value of one if a subject has a convex utility function and zero otherwise, and “Linear” a binary variable that takes a value of one if a subject has a linear utility function and zero otherwise. Robust standard errors in parenthesis. \*\*\* denotes significance at the 0.01 level, \*\* denotes significance at the 0.05 level, \* denotes significance at the 0.1 level.

## Appendix F: Utility functions

This appendix investigates the properties of the elicited utility functions. Decision sets 1 to 6 of the second part of the experiment are designed to elicit the sequence of outcomes  $\{x_1, x_2, x_3, x_4, x_5, x_6\}$  for each subject. This elicited sequence has the relevant property that it ensures equally-spaced utility values, i.e.  $u(x_j) - u(x_{j-1}) = u(x_{j-1}) - u(x_{j-2})$ , allowing me to characterize a subject's preference over monetary outcomes by mapping each utility value,  $u(x_j)$  to the subject's stated preference  $x_j$ .

I focus on two properties of the utility function: the sign of the slope and the curvature. To that end, I construct two variables, the first variable is  $\Delta'_i := x_j - x_{j-1}$ , for  $j = 1, \dots, 6$  and the second is  $\Delta''_j := \Delta'_j - \Delta'_{j-1}$  for  $i = 2, \dots, 6$ . The sign of  $\Delta'_j$  as  $j$  increases determines the sign of the slope, i.e. whether a subject prefers larger monetary outcomes to smaller monetary outcomes. Similarly, the sign of  $\Delta''_j$  as  $j$  increases determines the utility curvature. For example, a subject with  $\Delta'_j > 0$  and  $\Delta''_j > 0$  for all  $j$  exhibits a preference for larger monetary outcomes and experiences smaller utility increments with larger monetary outcomes, this is equivalent to say that this subject has an increasing and concave utility function.

The first analysis focuses on classifications at the individual level. I classify subjects according to the curvature of their utility function. Since I have multiple observations for each subject and it was possible that subjects made mistakes, this classification is based on the sign of  $\Delta''_j$  with the most occurrence. Specifically, a subject with at least three negative  $\Delta''_j$ 's was classified as having a convex utility, a subject with at least three positive  $\Delta''_j$ 's had a concave utility and subject with three or more  $\Delta''_j$ 's had a linear utility. A subject with a utility function that cannot be classified as concave, convex, or linear, had a mixed utility. Furthermore, to statistically assess the sign of a  $\Delta''_j$ , I construct confidence intervals around zero. In particular, I multiply the standard deviation of each  $\Delta''_j$  by the factors 0.64 and  $-0.64$ . Thus, if  $\Delta''_j$  follows a normal distribution, 50% of the data should lie within the confidence interval.<sup>37</sup>

The data suggest that all subjects in the experiment exhibit an increasing sequence  $\{x_1, \dots, x_6\}$  which denotes, not surprisingly, a generalized preference for larger amounts of money. Table 9 presents the classification of subjects according to the curvature of their utility function. The data suggest that the majority of subjects exhibit linear utility functions.

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<sup>37</sup>More stringent confidence intervals were also used for the analysis. These confidence intervals were also constructed using the standard deviation of a  $\Delta''_j$  which was multiplied by different factors, such as 1 and  $-1$ , 1.64 and  $-1.64$ , and 2 and  $-2$ . The qualitative results of these analyzes are not different from the main result presented here that the majority of subjects exhibit a linear utility function. This is not surprising inasmuch as these confidence intervals are more stringent and yield less subjects classified as having a mixed utility function and more subjects exhibiting a linear utility function.

Specifically, 77% of the subjects have linear utility, while the rest of the subjects have mixed utility (13% of the subjects), and concave utility (7% of the subjects). A proportions test suggest that the proportion of subjects with linear utility is significantly larger than 50% ( $p < 0.001$ ). Moreover, this test also yields that the proportion of subjects having linear functions is significantly larger than the proportion of subjects with mixed ( $p < 0.001$ ) and concave utility ( $p < 0.001$ ).

The result that more than two-thirds of the subjects exhibit linear utility is at odds with the principle of diminishing sensitivity, a key property of cumulative prospect theory (CPT). However, disregarding CPT as a possible representation for the subjects' preferences for money on the basis of this classification may be incorrect. As pointed out by [Wakker and Deneffe \(1996\)](#), their trade-off method, used to elicit  $\{x_1, x_2, x_3, x_4, x_5\}$ , requires lotteries with large monetary outcomes in order to obtain utility functions with curvature. Therefore, one of the advantages of the experimental design, that it elicits the utility function and the probability weighting function of subjects using monetary stakes that reflect the monetary incentives in the first part of the experiment, is also the reason that diminishing sensitivity is not be observed.

Table 9 also presents the results of the aforementioned analysis when it is assumed that subjects have CPT preferences with a reference point equal to the monetary equivalent of a subject's beliefs about his performance in the first part of the experiment. Monetary outcomes above this reference point are considered gains and outcomes below the reference point are considered losses. This alternative analysis also leads to the conclusion that the majority of the subjects exhibit a linear utility function. Specifically, I find that 65 % of the subjects have linear utilities in the domain of gains and 98% of the subjects exhibit linear utilities in the domain of losses.

To understand how the aforementioned results aggregate, I analyze the sequence  $\{x_1, \dots, x_6\}$  when each outcome  $x_j$  is averaged for all subject. Table 10 presents the descriptive statistics of the resulting outcomes. I find that the average outcome  $x_j$  is increasing with  $j$ , implying that on average subjects exhibit a taste for larger monetary outcomes. Moreover, the column displaying the average values of the variable  $\Delta'_j$  shows that as  $j$  increases, increments of  $x_j$  become larger. Thus, while on average subjects exhibit linear utility, this tendency ceases as monetary outcomes in the lotteries become larger. In fact, for large values of  $x_j$  the average utility function displays concavity. This result is also found by [Abdellaoui \(2000\)](#).

The last analysis of the data consists on fitting well-known parametric families of utility functions. Specifically, I assume a power utility, belonging to the CRRA family of utility functions, and an exponential function, belonging to the CARA family of utility functions. Table 11 the regression estimates when non-linear least squares is used to fit the data to the

Table 9: Classification of subjects according to utility curvature

Reference Point	Domain	Convex	Concave	Linear	Mixed	Total
No/Zero	No/Gains	3	13	133	23	172
Belief	Gains	3	12	43	21	79
Belief	Losses	0	1	90	2	93

Note: This table presents the subjects classification according to the shape of their utility function. Subjects are classified as having a convex, concave, linear or mixed utility function based on the sign of  $\Delta_j$  with more occurrence. The first row presents the classification when the analysis is performed with all the data. The second and third columns feature the analysis taking into account the Beliefs non-zero reference point, which is the monetary equivalent of a subject’s beliefs about her performance in the first part of the experiment. The second row presents the analysis when the monetary outcomes of the lotteries are above the reference point, whereas the third row presents the analysis when the monetary outcomes of the lotteries are below the reference point

assumed utility function. For the two parametric specifications I find that the average utility function of the subjects is approximately linear. For instance, when the power utility function  $u(x) = x^\phi$  is assumed, the parameter attains a value of 0.995. This finding is consistent with the large proportion of subjects that were classified as having a linear utility function in the individual analysis and the modest increments that the averaged outcomes  $x_j$  exhibit as  $j$  increases presented in Table 10.

These analyses are also performed under the assumption that subjects have CPT preferences with a reference point equal to the monetary equivalent of the subject’s belief in the first part of the experiment. According to Table 10, subjects exhibit an average preference for larger monetary amounts in both domains. Also, the descriptive statistics suggest a decreasing tendency of the utility function to be linear as the outcome becomes larger in the domain of gains and lower in the domain of losses. The latter finding implies that in the domain of gains the average utility function tends to concavity, while in the domain of losses the function it tends to convexity. Furthermore, the data suggest that diminishing sensitivity manifests at different degrees across the domains, with subjects exhibiting more in the domain of gains. This difference is explained by fact that only positive outcomes were used to elicit the sequence  $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ . This leaves little room for subjects to exhibit as much sensitivity in the domain of losses as in the domain of gains. Note that I chose to elicit preferences using only positive outcomes since the second part of the experiment was designed to understand the subjects’ risk preferences over the monetary incentives at stake in the first part of the experiment. A more complete analysis of diminishing sensitivity across domains, and of risk preferences in general, requires lotteries featuring negative outcomes.

Table 10: Aggregate results  $x_1, x_2, x_3, x_4, x_5$ , and  $x_6$

$j$	$x_j$	$\Delta'_j$	$x_j$	$\Delta'_k$	$x_j$	$\Delta'_j$
1	2.579 (1.990)	1.579	3.761(4.037)	3.037	1.576 (0.548)	0.576
2	4.573 (4.445)	1.993	8.167 (5.226)	4.129	2.167(0.931)	0.590
3	6.684 (6.792)	2.110	12.545(7.564)	4.378	2.761(1.280)	0.593
4	9.179 (9.420)	2.495	17.812 (9.826)	5.266	3.515 (1.800)	0.754
5	11.773 (11.880)	2.594	23.156(11.598)	5.344	4.353 (2.589)	0.837
6	14.379 (14.418)	2.605	28.400 (13.608)	5.243	5.287 (3.727)	0.934
Ref.Point	No/Zero		Beliefs		Beliefs	
Domain	No/Gains		Gains		Losses	

Note: This table presents the average, standard deviations of the sequence  $x_1, x_2, x_3, x_4, x_5, x_6$  along with the difference  $\Delta'_j = x_j - x_{j-1}$ . Standard deviations are presented in parenthesis. Columns 2 and 3 present these statistics when all the data is taken into account. Columns 4, 5, 6, and 7, present these statistics when it is assumed that subjects make decisions around Beliefs as a reference point. Columns 4 and 5 present the mean and median of  $x_1, x_2, x_3, x_4, x_5, x_6$  along with  $\Delta'_j = x_j - x_{j-1}$  for values above Beliefs for each subject. Columns 6 and 7 present the mean and median of  $x_1, x_2, x_3, x_4, x_5, x_6$  along with  $\Delta'_j = x_j - x_{j-1}$  for values below Beliefs for each subject.

I also estimate the parameters of the utility function for each domain assuming a power or an exponential utility function. For the domain of losses, the estimated coefficients suggest approximate linearity, with an estimated coefficient  $\phi = 0.992$  when a power utility function is assumed. A similar result is found for the domain of losses, where the estimation yields  $\phi = 1.035$ .

All in all, the data suggest that subjects have linear utility functions. This finding is robust to the assumption that subjects have CPT preferences and the reference point is assumed to be their belief. This is not a surprising finding given the magnitude of the stakes used to elicit the subject's risk preferences. Furthermore, the conclusion that the utility function is linear implies that probability risk attitudes fully determine the risk attitudes. Implying that performance differences across treatments must be explained by probability distortions.

Table 11: Parametric estimates of average utility function

Exponential (CARA) $1 - \exp(-\gamma(x_{j-1} + \frac{\epsilon}{2}))$			
$\hat{\gamma}$	0.977 (0.001)	0.946 (0.001)	1.337 (0.001)
Adj. R <sup>2</sup>	0.922	0.887	0.303
N	1032	412	619
Power Utility (CRRA) $(x_{j-1} + \frac{\epsilon}{2})^\phi$			
$\hat{\phi}$	0.995 (0.001)	0.992 (0.001)	1.035 (0.007)
Adj. R <sup>2</sup>	0.925	0.971	0.756
N	1032	412	619
Ref.Point	No/Zero	Beliefs	Beliefs
Domain	No/Gains	Gains	Losses

Note: This table presents the estimates of the non linear least squares regression. The upper panel assumes that the parametric form  $1 - \exp(-\gamma(x_{j-1} + \frac{\epsilon}{2}))$  and the lower panel assumes the parametric form  $(x_{j-1} + \frac{\epsilon}{2})^\phi$ . The first column presents uses all the data. The second and third column present the data for the domain of gains and the domain of losses, respectively, when the reference point is Beliefs. Standard errors in parenthesis.

## Appendix G: Individual analysis of probability weighting functions

This appendix presents alternative analyses of the probability weighting functions. Decision sets 7 to 11 included in the second part of the experiment were designed to elicit the subjects' weighting functions. In the main body of the paper, I present parametric analyses of the average data. In this appendix, I present non-parametric analyses of these data performed at the individual level.

The first analysis classifies each subject according to the shape of the elicited probability weighting function and is based on [Bleichrodt and Pinto \(2000\)](#). There were five possible shapes of the probability weighting function. A subject could display a weighting function with either lower subadditivity (LS), upper subadditivity (US) or with both properties. These three properties result from comparing the behavior of the probability weighting function at extreme probabilities to the behavior of the same function at intermediate probabilities. Moreover, a subject could display a concave or a convex probability weighting function.

To classify a subject into one of these five categories, I created the variable  $\partial_{j-1}^j := \frac{w(p_j) - w(p_{j-1})}{w^{-1}(p_j) - w^{-1}(p_{j-1})}$ , which captures the average slope of the probability weighting function between probabilities  $j$  and  $j - 1$ . I also created the variable  $\nabla_{j-1}^j \equiv \partial_{j-1}^j - \partial_{j-2}^{j-1}$ , which represents the change of the average slope of the weighting function between successive probabilities.

To understand the subjects' behavior at extreme and intermediate probabilities I focus on the sign of the variables  $\nabla_{0.16}^{0.33}$  and  $\nabla_{0.83}^1$ . If a subject exhibits  $\nabla_{0.16}^{0.33} < 0$ , his probability weighting exhibits LS. In other words, his probability weighting function assigns larger weights to small probabilities than to medium-ranged probabilities. Moreover, if a subject has  $\nabla_{0.83}^1 > 0$ , then his probability weighting function exhibits the property of US. That is, his weighting function assigns larger weights to large probabilities than to medium-ranged probabilities. The resulting dummy variables LS and US or Both were used in the main body of the paper to investigate the effect of these properties of the weighting function on the treatment effects.

In addition, I examine the sign of  $\nabla_{j-1}^j$  as  $j$  increases to determine the shape of the weighting function of each subject over the whole probability interval. A subject was classified as having a concave weighting function if at least three (out of five)  $\nabla_{j-1}^j$  had a negative sign and he did not exhibit US. Alternatively, a subject had a convex probability weighting function if at least three (out of five)  $\nabla_{j-1}^j$  were positive and he did not exhibit LS. Note that these classifications allow for the possibility of response error.

Table 12 presents the results of the individual classification. I find that 57 % of subjects exhibit LS, 75% of subjects exhibit US and 44% of subjects display probability weighting

functions with both LS and US. Therefore, most subjects in the experiment had weighting functions that yield overweighting of small probabilities or underweighting of large probabilities. Also, almost half of subjects exhibit probability weighting functions that assign large weights to small and large probabilities. These proportions are however considerably lower than those reported by [Bleichrodt and Pinto \(2000\)](#). Moreover, I find that 39% of the subjects exhibit convex weighting functions and only 13% of the subjects exhibit concave weighting functions. Thus, more subjects in the experiment exhibit pessimism. Furthermore, the proportion of subjects in the experiment with either concave or convex probability weighting functions is higher than that reported by [Bleichrodt and Pinto \(2000\)](#), who finds that only 15% of the subjects have probability weighting functions with either of these shapes.

Table 12: Classification of subjects according to the shape of their weighting function

Reference Point	Domain	Convex	Concave	LS	US	LS & US
No/Zero	No/Gains	68	23	98	129	76
Beliefs	Gains	29	9	49	63	38
Beliefs	Losses	39	14	49	66	38

Note: This table presents the classification of subjects according to the shape of their probability weighting function. Subjects are classified as having a probability weighting function with US, LS or both. Also, subjects are classified as having a convex or concave probability weighting function if they do not exhibit LS and US, respectively. This classification depends on the sign of  $\nabla_{j-1}^j$ . The first row presents the classification with all the data. The second and third columns feature the analysis assuming that the monetary equivalent of a subject belief in the real-effort task is the reference point. The second row presents the analysis when the monetary outcomes of the lotteries are above the reference point. The third row presents the analysis when the monetary outcomes of the lotteries are below the reference point.

For the sake of robustness, I perform an alternative classification of LS and US also proposed by [Bleichrodt and Pinto \(2000\)](#). In comparison to the above classification, weights given to extreme probabilities are contrasted to the corresponding objective probability. In particular a subject has a weighting function with LS if  $w^{-1}\left(\frac{1}{6}\right) < 0.16$ . Similarly, a subject has a weighting function with US if  $1 - w^{-1}\left(\frac{5}{6}\right) < 0.16$ . This alternative classification of LS and US is admittedly less accurate. The reason is that assigning large weights to extreme probabilities does not guarantee that the weights assigned to medium-ranged probabilities are small.

The results of the alternative classification are presented in [Table 13](#). I find that a similar proportion of subjects exhibit US and LS. Specifically, 40.12% of subjects exhibit LS and 38.37% subjects exhibit US. Also, only 20% of subjects exhibit both LS and US. These proportions are considerably lower than those obtained with the initial classification and are

also smaller to those reported by [Bleichrodt and Pinto \(2000\)](#).

Table 13: Classification of subjects according to LS, US, or both

Reference Point	Domain	LS	US	Both
No/Zero	No/Gains	55	89	25
Beliefs	Gains	18	49	8
Beliefs	Losses	37	40	17

Note: This table presents the classification of subjects according to the shape of their weighting functions. Subjects are classified as having weighting functions with LS if  $w^{-1}(\frac{1}{6}) < w^{-1}(\frac{2}{6}) - w^{-1}(\frac{1}{6})$ . Subjects have weighting functions with US if  $1 - w^{-1}(\frac{5}{6}) < w^{-1}(\frac{5}{6}) - w^{-1}(\frac{4}{6})$ . When these two properties hold, subjects are classified in Both.

The last considered classification, evaluates the strength of the possibility effect relative to the certainty effect. A subject exhibits a weighting function with a possibility effect that is stronger than the certainty effect when  $1 - w^{-1}(\frac{5}{6}) > w^{-1}(\frac{1}{6})$ . Table 14 shows that the majority of subjects in the experiment have probability weighting functions with the certainty effect exceeding the possibility effect. This result is in line with the findings of [Tversky and Fox \(1995\)](#). Nevertheless, the proportion of subjects for which Certainty exceeds Possibility is not negligible as it constitutes close to 32 % of subjects.

Table 14: Classification of subjects according to strength of possibility effect

Reference Point	Domain	Certainty	Possibility	Equal
No/Zero	No/Gains	107	55	10
Beliefs	Gains	57	18	4
Beliefs	Losses	50	37	6

Note: This table presents the classification of subjects according to the strength of the possibility effect with respect to the certainty effect. Subjects are classified Possibility, that is having probability weighting function where the possibility effect exceeds the certainty effect if  $1 - w^{-1}(\frac{5}{6}) > w^{-1}(\frac{1}{6})$ . Instead, if  $1 - w^{-1}(\frac{5}{6}) < w^{-1}(\frac{1}{6})$  subjects were classified certainty. Finally, subjects with  $1 - w^{-1}(\frac{5}{6}) = w^{-1}(\frac{1}{6})$  were classified Equal.

As in the main body of the paper, I consider the possibility that subjects have CPT preferences with a reference point equal to the monetary equivalent of their beliefs about performance in the first part of the experiment. All previous analyses are also performed under the assumption that the monetary equivalent of a subject's belief in the real-effort task

is the reference point.<sup>38</sup> The results of these analyses are also presented in Table 12, Table 13, and Table 14. All in all, I find that the aforementioned results are robust to subjects having CPT preferences. In particular for both domains there is a large proportion of subjects with US and/or LS. Also, regardless of the domain, more subjects exhibit weighting functions with the certainty effect being stronger than the possibility effect.

In conclusion, the analyses of the data at the individual level suggest that the majority of subjects have weighting functions with US or LS. Moreover, I find that less than half of subjects exhibit both properties at the same time, which is a remarkable difference with respect to Bleichrodt and Pinto (2000). Finally, as in Abdellaoui (2000) and Tversky and Fox (1995), I find that the certainty effect is stronger than the possibility effect for a larger share of individuals.

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<sup>38</sup>It is important to emphasize that the nature and intuition of the classification under the assumption that Beliefs is the reference point differs from the original classification. The reason for this difference is that the data does not admit enough  $\nabla_{j-1}^j$ s to analyze the shape of the probability weighting function of a subject for the domain of gains as well as for the domain of losses. Instead, I analyze the shape of a subject's probability weighting function for the domain wherein the majority of his  $\nabla_{j-1}^j$ s lie. Thus, this analysis could shed light on whether subjects who have most of their choices in the domain of losses exhibit weighting functions of different shape than subjects who have most of their choices in the domain of gains.

## Appendix H: Additional analyses

Table 15: Parametric estimates of the weighting function

	(1)	(2)	(3)
Panel 1: Neo-additive (truncated)			
$w(p) = c + sp$			
$\hat{c}$	0.194 *** (0.021)	0.228 *** (0.024)	0.155 *** (0.024)
$\hat{s}$	0.566 *** (0.035)	0.463 *** (0.037)	0.686 *** (0.044)
Log-Likelihood	220.288	75.200	166.842
Panel 2: Tversky & Kahneman (1992)			
$w(p) = \frac{p^\psi}{(p^\psi + (1-p)^\psi)^{\frac{1}{\psi}}}$			
$\hat{\psi}$	0.598 *** (0.016)	0.597 *** (0.012)	0.785 *** (0.037)
Adj. R <sup>2</sup>	0.838	0.827	0.866
Panel 3: Goldstein and Einhorn (1987)			
$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$			
$\hat{\gamma}$	0.281 *** (0.025)	0.196 *** (0.027)	0.426 *** (0.042)
$\hat{\delta}$	0.921 *** (0.020)	0.892 *** (0.029)	0.982 *** (0.032)
Adj. R <sup>2</sup>	0.863	0.845	0.888
Panel 4: Prelec (1998)			
$w(p) = \exp(-\beta(-\ln(p))^\alpha)$			
$\hat{\alpha}$	0.284 *** (0.025)	0.143 *** (0.025)	0.357 *** (0.033)
$\hat{\beta}$	0.841 *** (0.015)	0.596 *** (0.024)	0.944 *** (0.019)
Adj. R <sup>2</sup>	0.864	0.907	0.851
N	860	304	550
Ref.Point	No/Zero	Beliefs	Beliefs
Domain	No/Gains	Gains	Losses

Note: This table presents the estimates of the probability weighting function when parametric estimates are assumed. Panel 1 presents the maximum likelihood estimates of the equation  $w(p) = c + s(p)$  when truncation at  $w(p) = 0$  and at  $w(p) = 1$  is assumed. Panel 2 presents the non-linear least squares estimation of the function  $w(p) = \frac{p^\psi}{(p^\psi + (1-p)^\psi)^{\frac{1}{\psi}}}$ . The third panel presents the non-linear least squares estimates of the parametric form  $\frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$ . The last panel presents the non-linear least squares estimates of the function  $w(p) = \exp(-\beta(-\ln(p))^\alpha)$ . The first column in all the panels presents the estimates when all the data are used. The second and third columns present the estimations when it is assumed that Beliefs is the reference point and only data for the domain of gains and the domain of losses, respectively, is used for the estimations. Robust standard errors in parenthesis. \*\*\* denotes significance at the 0.01 level, \*\* denotes significance at the 0.05 level, \* denotes significance at the 0.1 level.